APPLICATION OF KALMAN FILTER TO OVERSAMPLED DATA FROM GLOBAL POSITION SYSTEM

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Abstract: There are many applications using the Global Positioning System (GPS) as a source of position and velocity data. A main problem of the GPS data is its inconsistency on higher sampling frequencies. The usage of enhanced Kalman Filter is presented to fill the gaps between the GPS data samples. The presented Kalman filter is enhanced to contain the velocity data on the output. The output velocity data meet the condition of integration and differentiation between the values of the position and the velocity.

Keywords: Kalman Filter, Oversampled Data, GPS, Flight Path Reconstruction, Aircraft

1 INTRODUCTION

There are many technologies based on the Global Position System (GPS). They use the GPS as a source of position and velocity with respect to the Earth. However, the GPS has a fairly low sampling frequency. Therefore it is needed to use sophisticated filtering algorithms to fill the gaps between the GPS data samples.

The measured data from a flight data acquisition system usually contain the GPS as global information source of position and velocity with respect to the Earth. It is important to filter this raw measured data to enhance its information consistency. This filtration step is used in the Flight Path Reconstruction (FPR). One of the best–known algorithms for the FPR is the Kalman Filter.

There are three main algorithms to filter nonlinear differential–equation–based systems. Gross et al. described in his summarizing article [1] Extended Kalman Filter, Unscented Kalman Filter and Particle Filter. By his experiments, it is clear that Extended Kalman Filter have greatest ratio between precision and computational cost.

Particle Filter, instead of Extended Kalman Filter, uses a random number generator to simulate the noise in the measurement. This approach simplifies the assumptions, but the computational cost is raising rapidly and the precision raising slowly[1].

Unscented Kalman Filter (UKF) is a type of a Kalman Filter [4]. UKF avoids the use of the Jacobian matrix of uncertainty, which is a major problem for the EKF. UKF realizes modeling of uncertainty by manipulation with sigma points [2]. This approach slightly increases the computational cost in the comparison with EKF.

This paper presents the enhancement of the Kalman Filter. The enhanced Kalman Filter enables a velocity data output based on the numerical differentiation. This enhancement of the Kalman Filter could be helpful to reduce artifacts in real–world applications.

This filtering step is a prerequisite for the visualization of the position in the modern flight deck instruments onboard of the general aviation aircraft. This filtered data could be the source for a later processing or could be used in the identification of flight parameters.

This paper is organized as follows. The following section shows the construction of the Kalman Filter. The model of system is described in section 3. The section 4 presents the enhanced Kalman Filter. The test results are presented in section 5.

2 KALMAN FILTER

The definition of continuous Kalman Filter is taken from [3]:

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{w} \tag{1}$$

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{2}$$

where $\dot{\mathbf{x}}$ is a vector of a time derivative of the position, $\mathbf{z}$ is a vector of the measured variables, $\mathbf{F}$, $\mathbf{D}$, and $\mathbf{H}$ are matrices of coefficients, $\mathbf{w}$ is a vector of a process noise and $\mathbf{v}$ is a vector of a measurement noise.

It is possible to use linear equations to describe the system [5]:

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{u}(k-1) + \mathbf{w}(k-1) \tag{3}$$

$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{v}(k) \tag{4}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{H}$ are matrices of coefficients.

The Kalman Filter consists of two steps: a prediction step and a correction step. The prediction step is defined by following equations:

$$\tilde{\mathbf{x}}(k+1) = \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k) \tag{5}$$

$$\tilde{\mathbf{P}}(k+1) = \mathbf{A}\hat{\mathbf{P}}(k)\mathbf{A}^T + \mathbf{Q} \tag{6}$$

where $\tilde{\mathbf{x}}$ is an estimated value of $\mathbf{x}$, $\hat{\mathbf{x}}$ is a corrected estimation of value $\mathbf{x}$, $\tilde{\mathbf{P}}$ is an estimated error covariance matrix, $\hat{\mathbf{P}}$ is a corrected estimation of the error covariance matrix, $\mathbf{Q}$ is a covariance matrix of the process errors and $\mathbf{A}^T$ is a transposition of the matrix $\mathbf{A}$.

The initial conditions are $\hat{\mathbf{x}}(0) = \mathbf{x}(0)$, $\hat{\mathbf{P}}(0) = \mathbf{P_0}$. $\mathbf{A}$, $\mathbf{B}$ after the expansion in Taylor series are equal to [3]:

$$\mathbf{A} = e^{\mathbf{F}\Delta t} \approx \mathbf{I} + \mathbf{F}\Delta t + \mathbf{F}^2\frac{\Delta t^2}{2!} + \dots \tag{7}$$

$$\mathbf{B} = \mathbf{D}\int_0^{\Delta t} e^{\mathbf{F}\tau} d\tau \approx \mathbf{D}\Delta t + \mathbf{D}\mathbf{F}\frac{\Delta t^2}{2!} + \mathbf{D}\mathbf{F}^2\frac{\Delta t^3}{3!} + \dots \tag{8}$$

where $\mathbf{I}$ is an identity matrix and τ is the integration step.

The extended Kalman Filter uses the equations (7) and (8) with just first elements of Taylor series. This step simplifies the computing in the nonlinear Kalman Filtering.

The correction step is defined by following equations:

$$\mathbf{K}(k) = \tilde{\mathbf{P}}(k)\mathbf{H}^T[\mathbf{H}\tilde{\mathbf{P}}(k)\mathbf{H}^T + \mathbf{R}]^{-1} \tag{9}$$

$$\hat{\mathbf{x}}(k) = \tilde{\mathbf{x}}(k) + \mathbf{K}(k)\{\mathbf{z}(k) - \mathbf{H}\tilde{\mathbf{x}}(k)\} \tag{10}$$

$$\hat{\mathbf{P}}(k) = \{\mathbf{I} - \mathbf{K}(k)\mathbf{H}\}\tilde{\mathbf{P}}(k) \tag{11}$$

where $\mathbf{K}$ is a Kalman gain matrix, $\mathbf{H}^T$ is a transposition of the matrix $\mathbf{H}$ and $\mathbf{R}$ is a covariance matrix of measurement errors.

The UKF uses slightly different equations. See [4] for further details.

3 THE MODEL OF THE SYSTEM

The system could be described with a help of a point near the Earth surface with the position and the velocity derived with respect to the Earth. The input data describing the system are based on the GPS measurements:

$$\mathbf{z} = \begin{bmatrix} \ell & \lambda & h \end{bmatrix}^T \tag{12}$$

$$\mathbf{u} = \begin{bmatrix} v_\ell & v_\lambda & v_h \end{bmatrix}^T \tag{13}$$

where $\mathbf{x}$ is a position vector, ℓ is the terrestrial latitude, λ is the terrestrial longitude, h is the geodetic height over the geoid, $\mathbf{u}$ is a velocity vector, v_ℓ is the terrestrial latitude speed, v_λ is the terrestrial longitude speed and v_h is the vertical geodetic speed over the geoid.

The GPS date representation is based on a EGM96 geoid [6], which is de facto a WGS84 ellipsoid with a correction grid. The WGS84 ellipsoid has been used for simplification of our assumptions. The difference between the WGS84 and the EGM96 is less than 107 m [6] which is significantly less than a. The WGS84 is used for position transformation from the polar (terrestrial latitude and longitude) coordinates to the cartesian coordinates and for the definition of the radius of curvature in the prime vertical N is represented by the equation [6]:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \ell}} \tag{14}$$

where a is a semi–major axis of the Earth (by the definition of WGS84, it is equal to $6,378,137.0\,m$) and e is the first eccentricity of the Earth which is defined by following equation [6]:

$$e = \sqrt{\frac{2}{1/f} - \frac{1}{(1/f)^2}} \tag{15}$$

where $1/f$ is a flattening (it is equal to 298.257223563 by the definition of WGS84).

The continuous model of the system can be described by a following set of differential equations:

$$\dot{\ell} = v_\ell (a + h) \tag{16}$$

$$\dot{\lambda} = v_\lambda (N + h) \tag{17}$$

$$\dot{h} = v_h \tag{18}$$

where $\dot{\ell}$ is a time derivative of the terrestrial latitude, $\dot{\lambda}$ is a time derivative of the terrestrial longitude, $\dot{h}$ is a time derivative of the geodetic height over the geoid.

In this model, $\mathbf{F}$ matrix is zero matrix and $\mathbf{H}$ matrix is identity matrix. This makes matrices $\mathbf{A}$ and $\mathbf{B}$ much simpler and allows us to express them as:

$$\mathbf{A} = \mathbf{I} \tag{19}$$

$$\mathbf{B} = \mathbf{D}\Delta t \tag{20}$$

This simplification changes the Kalman Filter to a form, which is equal to the Extended Kalman Filter.

4 THE ENHANCED KALMAN FILTER

The Kalman Filter uses the velocity as input variable. The output of the Kalman Filter is the position vector. For our purposes, it is necessary to filter velocity as well. The numerical differentiation can produce the velocity data on the output. This solution could be used in the real–time applications. For this purposes, the usage of a causality system is needed. One of the numerical differentiation methods compliant with the causal system is the finite backward differentiation. This finite backward differentiation is described by an equation [7]:

$$\dot{\mathbf{z}}'(k) = \frac{\mathbf{z}'(k) - \mathbf{z}'(k-1)}{\Delta t} \tag{21}$$

The equation (21) in combination with the equations (16), (17), and (18) could be used for the enhancement of the Kalman Filter to reveal the velocity data from the filtered position data.

This extension of the Kalman Filter is highlighted by grey color on following figure:

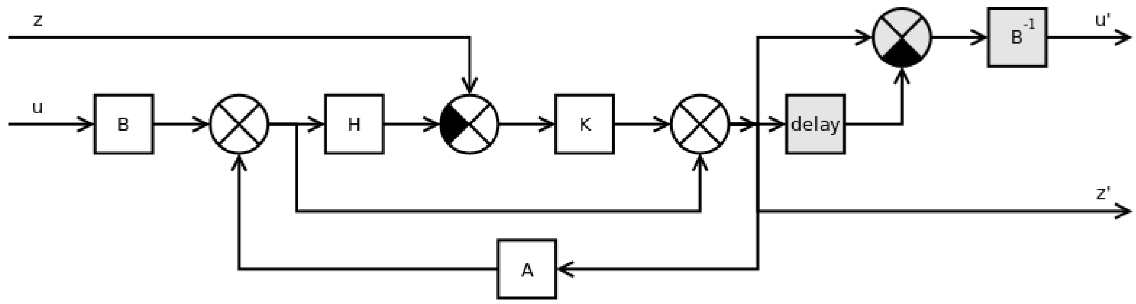
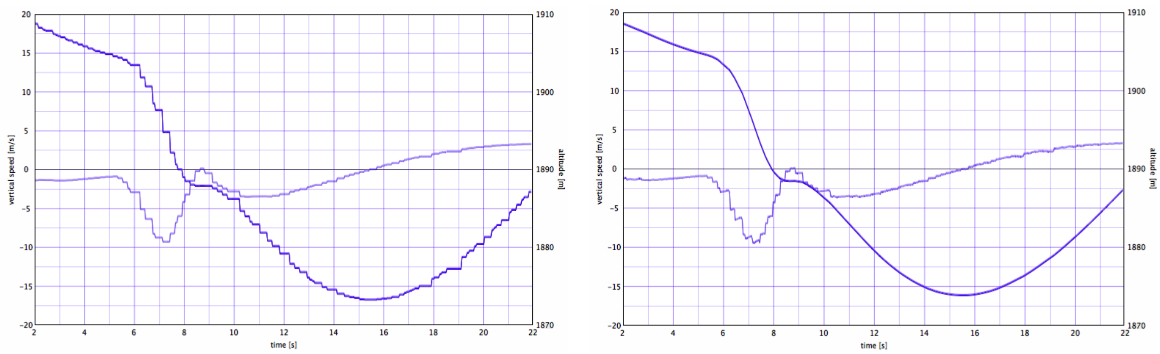


Figure 1: Schematic representation of the eKF. The grayed part is the added enhancement to the KF.

After the application of the enhanced Kalman Filter (eKF), the measured data fulfill the condition of the differentiation and integration, e.g. $\mathbf{v} \approx \dot{\mathbf{p}}$ and $\mathbf{p} \approx \int \mathbf{v}\, dt$.

5 RESULTS OF TESTS

The enhanced Kalman Filter has been tested on samples of the measured data.



(a) Raw measured data.

(b) Measured data after eKF.

Figure 2: Data before and after filtration by eKF.

In figure 2, the bold line represents the height above the mean sea level (altitude). The thin line represents the vertical speed, which is an altitude (position) time derivative. It is possible to see a significant advance of the position data quality and very small deformation of velocity data in figure 2b.

To decrease deformation, it is possible to use a dynamic change of the covariance matrix in order to reflect the frequency of updating of the measured samples from the GPS. The GPS has much lower frequency of updating with comparison to frequency of output data.

6 CONCLUSION

In this paper, the extension of the Kalman Filter is proposed to derive the velocity data from the filtered position data. The major advantage of the presented solution is the consistency between the position data and the velocity data.

Proposed enhanced Kalman Filter could be used for the enhancement of the consistency and the precision of the GPS data sampled on the higher frequencies. The presented approach could be used as an out-of-the-box solution. The filtered GPS data can then be used as an input for visualization in MEDS (Multifunction Electronic Display Subsystem) with higher frequency data for the smoother animation; or can decrease the lags of FMS (Flight Management System).

For the purposes of the performance evaluation, it is necessary to test the algorithm in real–time applications. For the future research, it is envisioned to measure the performance and precision changes from a dynamic extension of the Kalman Filter. This dynamic extension could decrease the deformation of the velocity data. The deformation of velocity intervention is the biggest disadvantage of presented algorithm.

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