FORWARD SCATTERING FROM 2D CYLINDERS WITH DIELECTRIC AND CONDUCTIVE PROPERTIES: A STEP TOWARD THE ELECTROMAGNETIC BREAST IMAGING

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Abstract: In context of the electromagnetic breast imaging, the scattering from cylinders that exhibit a contrast in electric permittivity and conductivity is analysed in a two-dimensional configuration. The analysis is carried out in the frequency-domain via integral and differential equation numerical approaches.

Keywords: forward scattering problem, integral equation technique, finite element method

1 INTRODUCTION

A reliable detection and identification of malignant human tissues have become an emergent chalenge during the last decades. One of the most important tasks in this respect is a development of diagnostic tool for an imaging of the breast cancer. Since the common X-ray mammography suffers from the important limmitations [1], a considerable effort has recently been invested in the development of microwave imaging systems [2, 3]. A first step toward the microwave imaging tool is an implementation of the efficient forward scattering solver [4].

As was experimentally proved, malignant tumors exhibit a contrast in electromagnetic properties with respect to a normal human tissue [5]. With these experimental results in mind, we compute the plane wave electromagnetic scattering from cylinders that show a dielectric and conductive contrast with respect to their embedding.

The first part of this paper is aimed at the description of the field problem connected with a TM (Transversal Magnetic) plane wave scattering from the cylinder in a two-dimensional configuration. Subsequently, we formulate the field equations that serve as a point of departure for further analysis. This part is divided into two parts describing (a) the equivalent contrast-source density formulation for an integral equation approach; (b) formulation for differential numerical techniques such as finite element method (FEM) [6]. Their numerical treatment is briefly decribed.

The analysis of the problem is carried out in the complex frequency-domain (s-domain). In conlusion we provide a number of numerical examples. The results from developed integral equation tool are compared with ones from the FEM-based solver.

2 PROBLEM DESCRIPTION

The problem configuration is given in Fig. 1. In it, the position is specified by the coordinates $\{x_1, x_2, x_3\}$ with respect to an orthogonal Cartesian reference frame with origin O and the three mutually perpendicular base vectors $\{i_1, i_2, i_3\}$ of unit length each. In the indicated order, the base vectors form a right-handed system.

The problem configuration consists of a scatterer (object) that is placed in a homogeneous, isotropic, lossless embedding characterized by its electric permittivity ε_0 and magnetic permeability μ_0. The corresponding wave speed is $c_0 = (\varepsilon_0\mu_0)^{-1/2}$ (real positive constant). The scattering object is homogeneous, isotropic and is described by its electric permittivity ε_1, magnetic permeability μ_0 and electric conductivity σ.

Throughout the paper, the subscript notation for Cartesian tensors is used and the summation convention for repeated subscripts applies [6, A.2] with Greek lower-case symbols that run over $\{1,2\}$. Partial differentiation with respect to x_m is denoted as ∂_m and symbol $e_{k,m,p}$ denotes the Levi-Civita tensor (completely anti-symmetric tensor of rank three) [6, A.7].

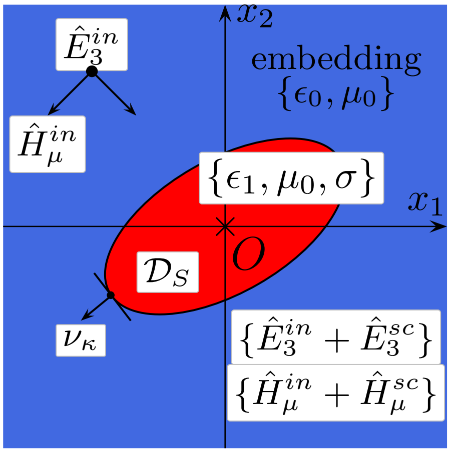


Figure 1: Problem configuration.

3 FORMULATION OF THE FIELD PROBLEM AND ITS NUMERICAL SOLUTION

In this section we describe the formulations of the forward scattering problem for the integral and differential equation numerical approaches [6, Sec. 28.9]. An analysis is carried out for the complex frequency-domain parameter s, with $\mathrm{Re}(s) > 0$. Subsequent numerical treatment is briefly discussed.

The scattering object is supposed to be irradiated by the TM-polarized plane wave

$$\hat{E}_3^{in}(x_\mu, s) = \hat{f}(s)\exp\{s[x_1\cos(\alpha) + x_2\sin(\alpha)]\} \tag{1}$$

where α is the incident angle with respect to x_1 axis and $\hat{f}(s)$ is the amplitude. The problem is to find the total field in the problem configuration. To this end, the total field is expressed as the sum of the incident (*in*) and the scattered field (*sc*) that accounts for the presence of the scatterer.

3.1 Integral equation approach

Considering the structure of the electromagnetic field equations, the scattered field in $\mathcal{D}_S$ can be found from

$$e_{\pi,\mu,3}\partial_\mu\hat{H}_\pi^{sc} + \hat{\eta}_0\hat{E}_3^{sc} = -(\hat{\eta} - \hat{\eta}_0)\hat{E}_3 \tag{2}$$

$$e_{\kappa,\mu,3}\partial_\mu\hat{E}_3^{sc} + \hat{\zeta}_0\hat{H}_\kappa^{sc} = 0 \tag{3}$$

where $\hat{\eta}_0 = s\varepsilon_0$, $\hat{\eta} = s\varepsilon_1 + \sigma$ and $\hat{\zeta}_0 = s\mu_0$. The right hand side of Eq. (2) is called as the equivalent contrast source (electric current) density. With the help of the fundamental solution of the wave equation (in s-domain), the scattered electric field strength can be written as

$$\hat{E}_3(x_\mu, s) + \frac{s\mu_0}{2\pi} \int_{x^s_\mu \in \mathcal{D}_S} \mathrm{K}_0[sr(x_\mu|x^s_\mu)](\hat{\eta} - \hat{\eta}_0)\hat{E}_3(x^s_\mu)\mathrm{d}A = \hat{E}^{in}_3(x_\mu, s) \tag{4}$$

for $x_\mu \in \mathbb{R}^2$. Here, $\mathrm{K}_0(x)$ denotes the modified Bessel function of the second kind and of order 0 and $r(x_\mu|x^s_\mu) = [(x_\mu - x^s_\mu)(x_\mu - x^s_\mu)]^{1/2} > 0$. Eq. (4) constitutes the Fredholm integral equation of the second kind that is solved numerically. At first, Eq. (4) is solved for unknown contrast sources for $x_\mu \in \mathcal{D}_S$. Once the equivalent contrast sources are known, the total electric field strength can be found by means of Eq. (4) for $x_\mu \in \mathbb{R}^2 \setminus \mathcal{D}^S$.

For a numerical solution, the surface of the scatterer is divided into a number of disjoint triangles over which the surface integrals are evaluated with the help of a numerical integration formula. On account of the weak (logarithmic) singularity of the corresponding Green's function, Cauchy's domain integral over a infinitesimal circular domain leads to a vanishing value. When the numerical integration is performed, the system of linear algebraic equations is obtained and solved.

3.2 DIFFERENTIAL EQUATION APPROACH

An usual point of departure for numerical discretization procedures is the following system of equations

$$e_{\pi,\mu,3}\partial_\mu \hat{H}^{sc}_\pi + \hat{\eta}\hat{E}^{sc}_3 = -(\hat{\eta} - \hat{\eta}_0)\hat{E}^{in}_3 \tag{5}$$

$$e_{\kappa,\mu,3}\partial_\mu \hat{E}^{sc}_3 + \hat{\zeta}_0\hat{H}^{sc}_\kappa = 0 \tag{6}$$

in $\mathcal{D}_S$. Elimination of magnetic field strengths yields

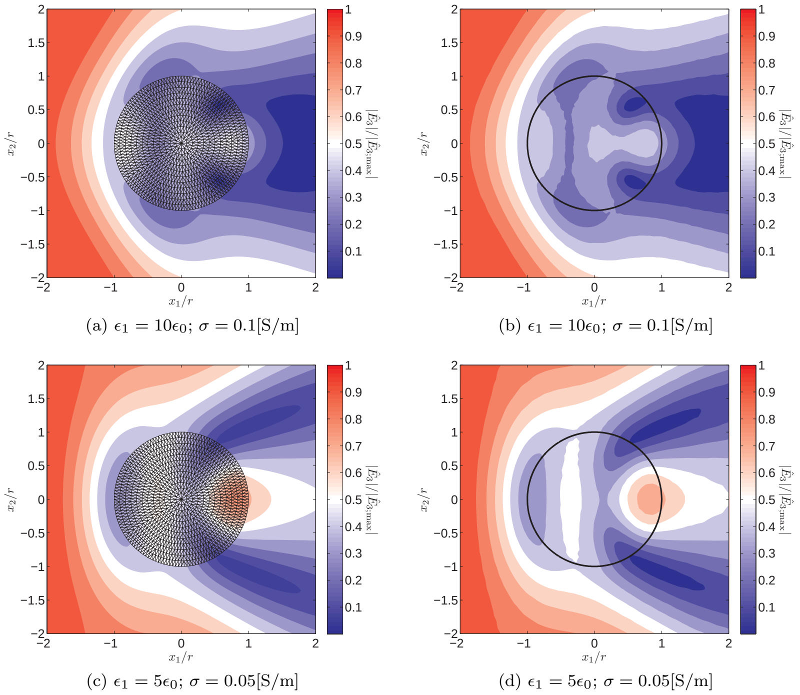
$$\partial_\pi\partial_\pi \hat{E}^{sc}_3 - \hat{\eta}\hat{\zeta}_0\hat{E}^{sc}_3 = \hat{\zeta}_0(\hat{\eta} - \hat{\eta}_0)\hat{E}^{in}_3 \tag{7}$$

in $\mathcal{D}_S$. Eq. (7) can be numerically solved by the finite element method [7]. Since the finite element method can only be applied to a bounded domain, the solution domain has to be truncated by an artificial boundary - perfectly matched layer (PML) that should minimize any nonphysical relflections (thought with respect to our configuration). Subsequently, the truncated domain is divided into a set of disjoint finite elements - usually triangles for two dimensional problems. Then, the electric field strength is approximated over each finite element using basis functions. Every basis function is associated with one edge of the finite element. Polynomials of various orders can be used as basis functions. Once the approximation is substituted into the solved equation the error function (called as a residuum) appears and is minimized by the Galerkin method [7]. In this manner we can arrive at the system of linear equations that is solved. Using the coefficients and basis functions the desired EM quantity can be interpolated in any position of the computational domain.

4 NUMERICAL RESULTS

This section provides a couple of numerical examples connected with the electromagnetic scattering from cylinders with circular and elliptical cross-sections. The results were computed with the help of the in-house integral equation (IE) solver implemented in Matlab and with FEM-based tool - Comsol Multiphysics. The latter serves mainly for the validation purposes.

In all analyses, the incident angle of the plane wave is taken as $\alpha = \pi$, the complex frequency-domain parameter is set to $s = 2\mathrm{i}\pi c_0/5r$ (frequency-domain analysis) and $\hat{f}(s) = 1$. Here, r denotes the radius of the circular cylinder and the length of the minor axis (for the elliptical cross-section). The length of the major axis is taken as $2r$. Two sets of electromagnetic properties of analyzed scatterers are considered: $\varepsilon_1 = 5\varepsilon_0, \sigma = 0.05$ [S/m] and $\varepsilon_1 = 10\varepsilon_0, \sigma = 0.1$ [S/m].



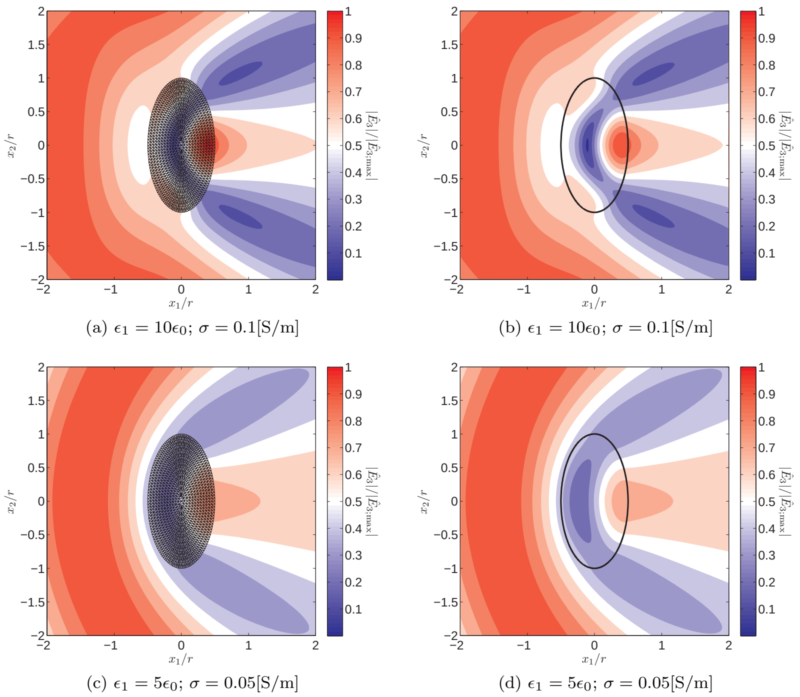
(a) $\epsilon_1 = 10\epsilon_0$; $\sigma = 0.1$[S/m]

(b) $\epsilon_1 = 10\epsilon_0$; $\sigma = 0.1$[S/m]

(c) $\epsilon_1 = 5\epsilon_0$; $\sigma = 0.05$[S/m]

(d) $\epsilon_1 = 5\epsilon_0$; $\sigma = 0.05$[S/m]

Figure 2: Absolute value of the normalized electric field strength distribution at $s = 2\mathrm{i}\pi c_0/5r$ for cylindrical scatterers with circular cross-section. Electromagnetic properties of the scattering objects are given below each figure. Subfigures (a) and (c) are evaluated using IE approach and (b) and (d) using FEM.



(a) $\epsilon_1 = 10\epsilon_0$; $\sigma = 0.1$[S/m]

(b) $\epsilon_1 = 10\epsilon_0$; $\sigma = 0.1$[S/m]

(c) $\epsilon_1 = 5\epsilon_0$; $\sigma = 0.05$[S/m]

(d) $\epsilon_1 = 5\epsilon_0$; $\sigma = 0.05$[S/m]

Figure 3: Absolute value of the normalized electric field strength distribution at $s = 2\mathrm{i}\pi c_0/5r$ for cylindrical scatterers with elliptical cross-section. Electromagnetic properties of the scattering objects are given below each figure. Subfigures (a) and (c) are evaluated using IE approach and (b) and (d) using FEM.

Figs. 2–3 reveal the results in the computational domain $\{-2 \leq x_1/r \leq 2,\ -2 \leq x_2/r \leq 2\}$. The normalized electric field strength $\hat{E}_3$ for the circular and elliptical scattering object is given in Fig. 2 and Fig. 3, respectively. The figures denoted by (a) and (c) show the results from the IE solver while (b) and (d) denote the results from the FEM solver. As can be seen, these results are almost identical. Considering the fact that the IE approach requires the discretization of the scatterer domain $\mathcal{D}_S$ only (see Figs. 2–3(a) and (c)), our IE solver is computationally less demanding.

5 CONCLUSIONS

The electromagnetic direct scattering problem for two-dimensional cylindrical structures with dielectric and conductive properties has been formulated and solved by means of the integral and differential approaches. The integral equation numerical approach have been implemented in Matlab and the obtained results have been confronted with the results from the FEM-based commercial software. The implemented integral equation solver can serve for the purpose of a development of microwave breast imaging system.

ACKNOWLEDGEMENT

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