MAGNETIC SUSCEPTIBILITY MODELING USING COMSOL

Michaela Burdková[1], Petr Marcoň[2]

Doctoral Degree Programme (1, 2), FEEC BUT

E-mail: [1]xburdk00@stud.feec.vutbr.cz, [2]xmarco00@stud.feec.vutbr.cz

Supervised by: Karel Bartušek

E-mail: bar@isibrno.cz

Abstract: The paper deals with modeling of the magnetic induction in close to the non-ferromagnetic materials. The nuclear magnetic resonance (NMR) based method for the material susceptibility measuring gives variable results which need to be verified by other reliable method. The COMSOL model based on Finite element method (FEM) makes the verification of measured results possible. Therefore, the use of this program is appropriate for the verification of the results. Paramagnetic and diamagnetic material samples e. g. platinum, aluminum, copper and bismuth are used. The samples are of cylinder and block shape. The length of the sample is 50 mm. The samples are placed in an air filled block.

Keywords: COMSOL, NMR, magnetic susceptibility, reaction field

1. INTRODUCTION

The magnetic susceptibility is the physical quantity describing material properties in the external magnetic field [1]. Magnetic susceptibility is defined as the ratio between magnetization $\mathbf{M}$ of the material in the magnetic field and the field intensity $\mathbf{H}$:

$$\mathbf{M} = \chi_m \mathbf{H}. \tag{1}$$

All materials can be classified by the value of magnetic susceptibility into three groups:

- diamagnetic materials: $-1 < \chi_m < 0$,
- paramagnetic materials: $0 < \chi_m \ll 1$,
- ferromagnetic materials: $\chi_m \gg 1$.

Several methods are used for the magnetic susceptibility measuring such as Faraday´s scale, Guoy's scale or inductive method with the SQUID magnetometer. For detailed description of these methods see [2]. The MR based method for susceptibility measurement makes also this measurement possible [3], [4]. Whereas, this method gives us inaccurate results, other method for results verification is needed. A possible option is to use the results of the COMSOL software simulations of the magnetic field close to the different materials for the magnetic susceptibility calculation. Placing the paramagnetic and the diamagnetic materials in the homogenous magnetic field cause some distortions,

which are corresponding to the magnetic susceptibility χ of the measured material. In this article modeling of this effect is discussed.

2. PROPERTIES AND ASSEMBLY OF THE EXPERIMENT

Space configuration of the sample is shown in the figure 1. Block space (50x50x80 mm) filled by air has magnetic susceptibility $\chi_{ml} = 0$, labeled by the number 1 in the figure 1. In the center of this space is placed the sample of mentioned materials with different magnetic susceptibility χ_m. The samples are in the shape of a cylinder and a prism with a length of 40 mm, labeled by the number 2 in the figure 1. The cylinder has a diameter of 5 mm and the prism has a square cross-section of the 2,5 mm. There were used the paramagnetic samples: aluminum ($\chi_{mAl} = 2,2 \cdot 10^{-5}$), platinum ($\chi_{mPt} = 2,65 \cdot 10^{-4}$) and the diamagnetic samples: copper ($\chi_{mCu} = -9,2 \cdot 10^{-6}$) and bismuth ($\chi_{mBi} = -1,66 \cdot 10^{-4}$). Section plane is placed to the center of the cubic space, labeled by the number 3 in the figure 1.
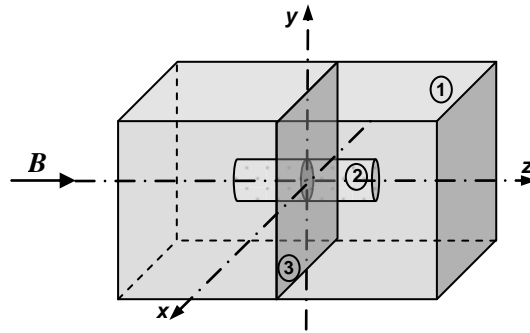

Figure 1: The configuration of the modeled system, cube filled by air with sample.

The model according to the arrangement shown in the figure 1 was designed in the COMSOL environment. The model was designed with the use of the finite element method. The boundary conditions were chosen according to the formula for the magnetic potential in order to the value of a homogenous static magnetic field induction was $B = 4,7$ T.

$$\pm \frac{\varphi}{2} = \frac{B \cdot z}{2\mu_0} = \frac{4,7\,[\text{T}] \cdot 80\,[\text{mm}]}{2 \cdot \mu_0}\,. \tag{2}$$

3. NUMERICAL MODELING

The analysis in the program COMSOL is based on Finite Element Method (FEM). For description of the quasi-stable model by the FEM are used the reduced Maxwell's equations:

$$rot\ \mathbf{H} = \mathbf{J},\ div\ \mathbf{B} = 0, \tag{3), (4}$$

where $\mathbf{H}$ is the magnetic field intensity vector, $\mathbf{B}$ is the magnetic field induction vector, $\mathbf{J}$ is the current density vector.

For case of the irrotational field is the equation (3) in the form:

$$rot\ \mathbf{H} = 0.$$

(5)

Material relations are given by equation:

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H}.$$

(6)

4. THE RESULTS OF NUMERICAL MODELING

The difference between the calculated value of the magnetic field B and the size of the static magnetic field B_0 is called the reaction field ΔB. The modeled reaction field ΔB of samples is shown in the section plane in figure . Value $\Delta B = B_{z\_emnc}$ - 4,7 is plotted on the y axis. The curves are plotted in the cut axis x in the middle section of the modeled sample (Figure 1).
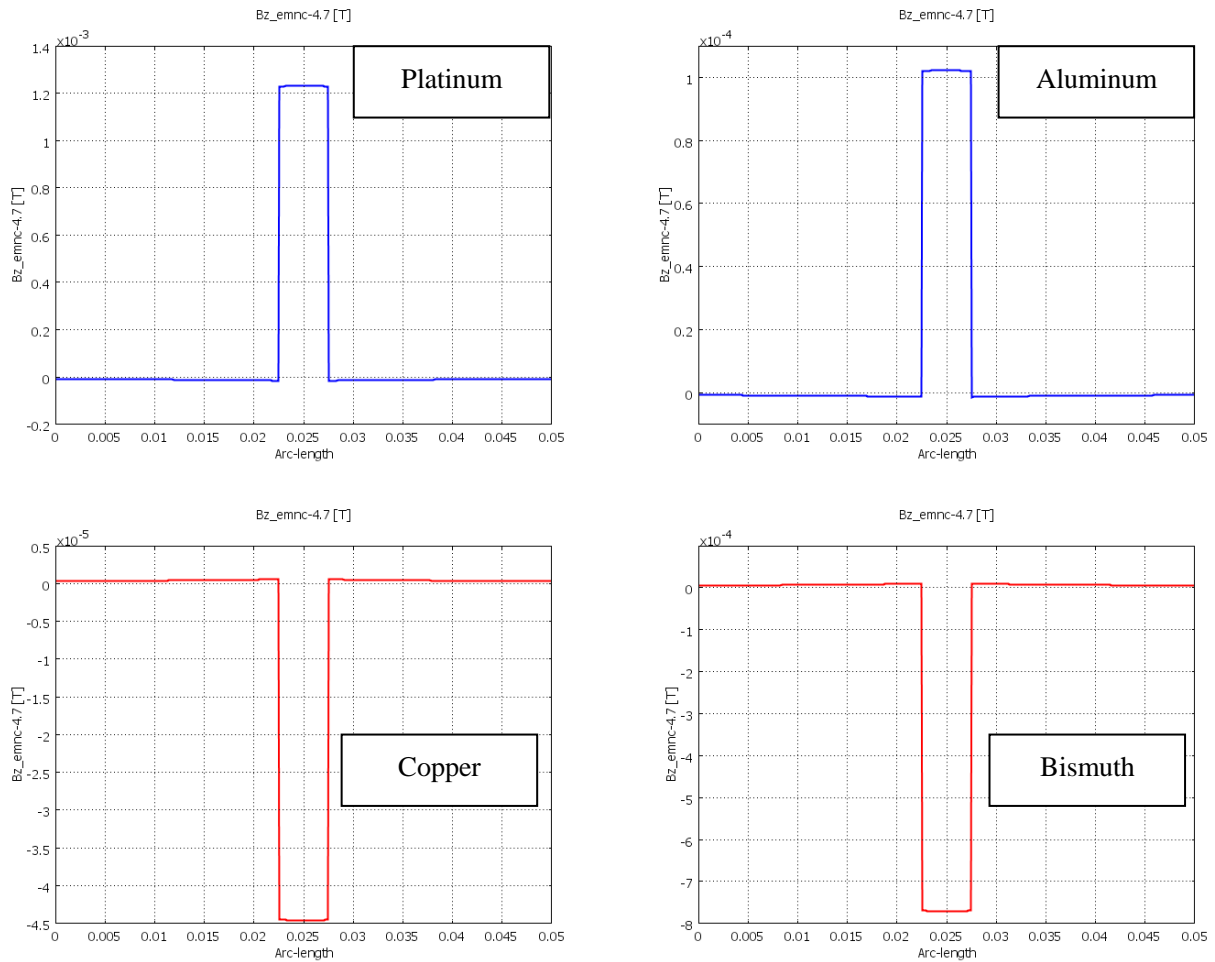

Figure 2: The courses of the reaction field in the axis x in the middle of the measured samples (cylinder).

To calculate the magnetic susceptibility is used the course of the reaction field in the axis x in the

middle of the modeled sample. The magnetic susceptibility of the modeled material is then equal to:

$$\chi_m = \pm \frac{B_{max} - B_{min}}{B_0}, \tag{7}$$

where the ± sign before the fraction depends on the used material. Sign (+) was given before fraction for calculation of the susceptibility of paramagnetic substances and sign (−) for calculation of the susceptibility of diamagnetic substances. Value B_{max} and B_{min} represent the stabile value of ΔB in space of the sample and the minimal B_{min} value just next to the material sample. By using reaction field ΔB values from the figure and the equation (7) the magnetic susceptibility of simulated samples is calculated. In the table is a summary of the results of the magnetic susceptibility of all the samples.
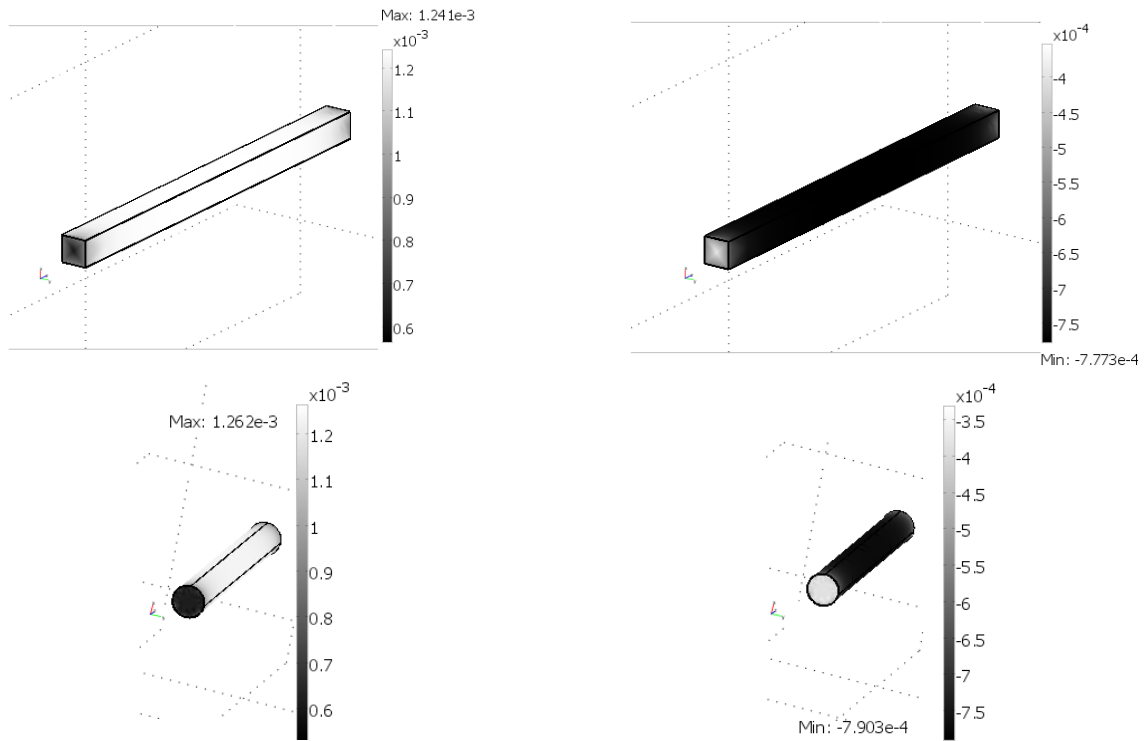

Figure 3: Distribution of reaction field ΔB on the surface of platinum prism and cylinder (left) and prism and cylinder of bismuth (right).

On the figure 3 is illustrated the distribution of reaction field on the surface samples. On the left is the platinum cylinder and the prism. From the left picture can be seen that the magnetic field inside the sample is stronger than the field B_0. Thus we have verified that it is a paramagnetic substance. Similar distribution of the reaction field (smaller change) has also the aluminum samples. The pictures on the right side of page are showing the prism and the cylinder of bismuth. The field inside the sample is smaller than the magnetic field B_0. Thus we have verified that this is a diamagnetic substance. Similar distribution of the magnetic field (smaller change) has also copper.

5. CONCLUSIONS

The COMSOL environment was used for the modeling of interactions of the magnetic field with paramagnetic and diamagnetic samples. Proposed equation (7) gives values of the magnetic susceptibility with sufficient accuracy. Higher level of the accuracy is possible with increasing of the element size. In the table 1 is shown an overview of the actual values of magnetic susceptibility χ of measured materials and realtive errors $\delta\chi$ calculated from the model of individual samples.

Samples	χ_t [ppm]	χ_m [ppm]	$\delta\chi$ [%]
Prism - Platinum	265,0	264,0	-0,36
Prism - Aluminum	22,0	22,3	1,55
Prism - Copper	-9,2	-9,17	-0,32
Prism - Bismuth	-166,0	-165,0	-0,37
Cylinder - Platinum	265,0	269,0	1,32
Cylinder - Aluminum	22,0	22,3	1,26
Cylinder - Copper	-9,20	-9,32	1,30
Cylinder - Bismuth	-166,0	-168,0	1,29

Table 1: Differences between the table and computed value of the magnetic susceptibility

It is possible to verify the experimental results of the magnetic susceptibility measuring for different material samples by modeling of the magnetic field. Application of equation (7) was verified by practical measurement of reaction field ΔB on the ÚPT AV ČR. The static magnetic field B_0 in the tomograph was 4,7 T. Aluminum and a copper cylinders were selected as the samples.

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