LPV APPROACH TO ROBUST CONTROL OF AC INDUCTION MOTOR

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Abstract: This paper deals with robust control of linear parameter varying (LPV) system (AC induction motor) by a LPV controller. State space equations of an AC induction motor in α-β stator fixed frame are nonlinear with respect to rotor speed. Nonlinear equations are rewritten to parameter form describing dependency on rotor speed which is assumed to be known by measurement or by estimation. LPV controller is designed using H_∞ theory to govern stator currents of an AC induction motor. Simulation results presented in this paper show that current controller provides good tracking and disturbance rejection over whole range of rotor speeds.

Keywords: LPV, gain scheduling, H_∞, AC induction motor

1. INTRODUCTION

Commonly used current control techniques for AC induction motor are treating nonlinear equations in two different ways, both based on the linearization of motor equations.

First approach uses Taylor Series expansion of a function around some point of interest. This method will result in equations which are locally linear. But more the trajectory deviates from the point of interest more the linearized model will differ from actual motor model. Controller derived from this linear representation will show different results in real application depending on state of the AC induction motor such as motor speed or parameter deviation. Robust methods can be used to find globally stabilizing controller for all speed and parameter variations but at the cost of controller performance.

Second approach uses exact linearization to find substitute which will make the motor equations linear and controllable. Unlike the first method exact linearization should provide minimal difference between linearized and actual model independently of current state. Unfortunately this method has one considerable disadvantage which is necessity to know the values of state space vector but the physical construction of ordinary induction motor does not allow to measure some of the state space variables (fluxes for example) and these need to be estimated from remaining variables.

Besides the control methods described above there are some nonlinear control techniques for controller synthesis but mathematical skills needed for stabilization of nonlinear plant with nonlinear controller are considerable. LPV method described in this article will provide linear state space equations which will require only rotor speed to be known.

2. AC INDUCTION MOTOR AS LINEAR PARAMETER VARYING SYSTEM

LPV system can be described by state space equations of the following form:

$$\dot{x} = A(\theta(t))x + B(\theta(t))u$$
$$y = C(\theta(t))x + D(\theta(t))u \tag{1}$$

There are at least two types of systems that can be described as an LPV system:

- Linear time invariant plants with time varying parametric uncertainty
- Nonlinear plats that can be linearized along the trajectories of some known parameters

Let's consider following state space equations of an AC induction motor in α-β stator fixed frame:

$$G: \begin{array}{rc} \dot{x}_1 = & a_1(x_2x_5 - x_3x_4) + a_2x_1 + a_3\tau_l \\ \dot{x}_2 = & a_4x_2 - n_px_1x_3 + a_5x_4 \\ \dot{x}_3 = & n_px_1x_2 + a_4x_3 + a_5x_5 \\ \dot{x}_4 = & a_6x_2 + a_7x_1x_3 - \gamma x_4 + a_8u_1 \\ \dot{x}_5 = & -a_7x_1x_2 + a_6x_3 - \gamma x_5 + a_8u_2 \\ y = & [x_1, x_4, x_5]^T \end{array} \tag{2}$$

With state vector $x = [\omega, \varphi_a, \varphi_b, i_a, i_b]^T$ and following parameters:

$$a_1 = \frac{n_pL_{sr}}{D_mL_r}, \quad a_2 = -\frac{R_m}{D_m}, \quad a_3 = -\frac{1}{D_m}, a_4 = -\frac{1}{T_r}, \quad a_5 = \frac{L_{sr}}{T_r},$$
$$a_6 = \frac{L_{sr}}{T_r\sigma L_sL_r}, a_7 = \frac{n_pL_{sr}}{\sigma L_sL_r}, \quad a_8 = \frac{1}{\sigma L_s}, \quad T_r = \frac{L_r}{R_r}, \tag{3}$$
$$\gamma = \frac{R_s}{L_s\sigma} + \frac{L_{sr}^2}{L_s\sigma L_rT_r}, \quad \sigma = 1 - \frac{L_{sr}^2}{L_sL_r}$$

If we take out first state variable ω, we will get nonlinear equations affinely dependent on only one parameter.

State space equations of an AC induction motor affinely dependent on ω:

$$\dot{x} = (A_0 + \omega A_1)x + Bu$$
$$y = Cx + Du \tag{4}$$

$$A_0 = \begin{bmatrix} a_4 & 0 & a_5 & 0 \\ 0 & a_4 & 0 & a_5 \\ a_6 & 0 & -\gamma & 0 \\ 0 & a_6 & 0 & -\gamma \end{bmatrix} A_1 = \begin{bmatrix} 0 & -n_p & 0 & 0 \\ n_p & 0 & 0 & 0 \\ 0 & a_7 & 0 & 0 \\ -a_7 & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ a_8 & 0 \\ 0 & a_8 \end{bmatrix} C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

Our goal is to design a LPV controller which will adjust to the plant dynamics based on information on ω - LPV controller will provide gain-scheduling with respect to varying parameter ω.

$$\dot{x}_k = A(\omega)x_k + B(\omega)y$$
$$u = C(\omega)x_k + D(\omega)y \tag{6}$$

However finding stabilizing controller for all admissible values of ω would be difficult because there is infinite number of stability conditions to verify. It is possible to reduce infinite number of solutions using vertex property [3] and design the controller at the vertices of given plant. Controllers at the vertices are designed offline and their impact on control action is based on the varying parameter.

3. H_∞ LOOP SHAPING

In order to stabilize the closed loop for all variations of ω it is necessary to satisfy following conditions at the vertices of the given plant:

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I + GK)^{-1} [I \quad G] \right\|_\infty < \gamma \tag{7}$$

Where $\gamma > \gamma_{opt}$, $1/\gamma_{opt}$ represents the maximum robust stability margin[4]. It is possible to specify desired closed loop performance by using weighting functions. Position of weighting function in closed loop system will affect the shape of each related system function defined in equation (7). Each system function represents one property of conventional closed loop system, for example $(I + GK)^{-1}$ is sensitivity function and it relates to disturbance rejection and its counterpart the complementary sensitivity function $GK(I + GK)^{-1}$ relates to tracking performance. Remaining two system functions are related to plant input error rejection and control energy reduction. Only two of the total four system functions were considered for H_∞ loop shaping of LPV current controller:

$$\left\| \begin{matrix} W_s & S \\ W_T & T \end{matrix} \right\|_\infty < \gamma \tag{8}$$

Where $S = (I + GK)^{-1}$ is the sensitivityfunction and $T = GK(I + GK)^{-1}$ is the complementary sensitivity function.

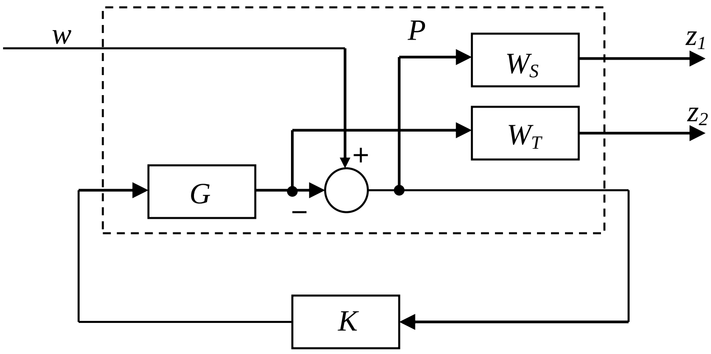

Fig.1. Augmented closed loop

Weight w_S was selected as $\frac{0.5s+550}{s+1.1}$so that sensitivity function will have character of a high pass filter. It is desired to keep the values of sensitivity function frequency response less than 1 (0dB, $1/w_S$) to highest frequency possible in order to guarantee rejection of disturbances. For reference tracking the complementary sensitivity function is shaped with static function with a value of 0.8 this will keep frequency response above 1 ($1/w_T$). These two requirements cannot be both satisfied simultaneously; therefore feedback controller design is trade-off over frequency of conflicting objectives. H_∞ suboptimal controller is computed by solving linear matrix inequality for the closed loop system [3], minimizing closed loop quadratic H_∞ performance from input *w* to output *z* (Fig.1). LPV controller can be computed using Matlab function hinfgs from Robust Control Toolbox, this function requires augmented closed loop system (it can be constructed using Matlab function sconnect) with all weighting functions included as shown in Fig.1. This function returns stabilizing controller K_{LPV}, achieved γ and closed loop systems R, S containing controller K_{LPV}. LPV current controller performance for unit step response with evenly spaced values of ω is shown in Fig. 3.

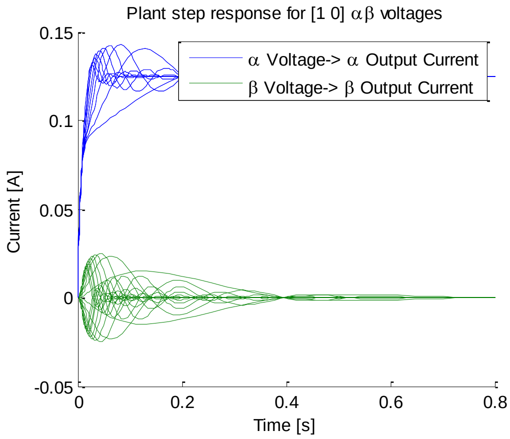

Fig. 2. Unit step response of a plant for different values of ω

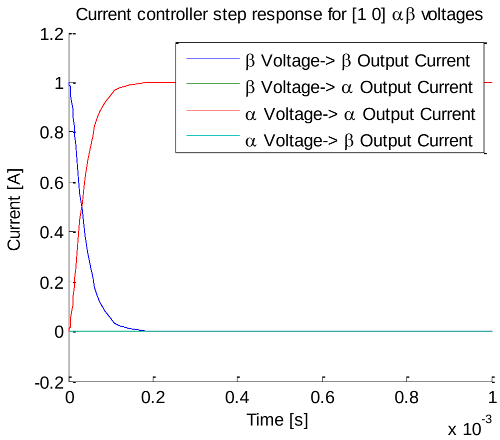

Fig. 3. Unit step response of the current closed loop for different values of ω

Overall performance of the current feedback loop was tested with a simple static controller which provides tracking of desired speed and flux of the linearized plant.

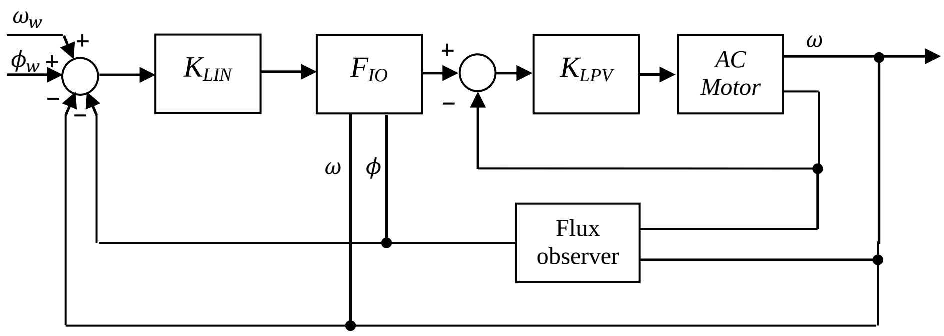

Fig.4. Speed controller structure

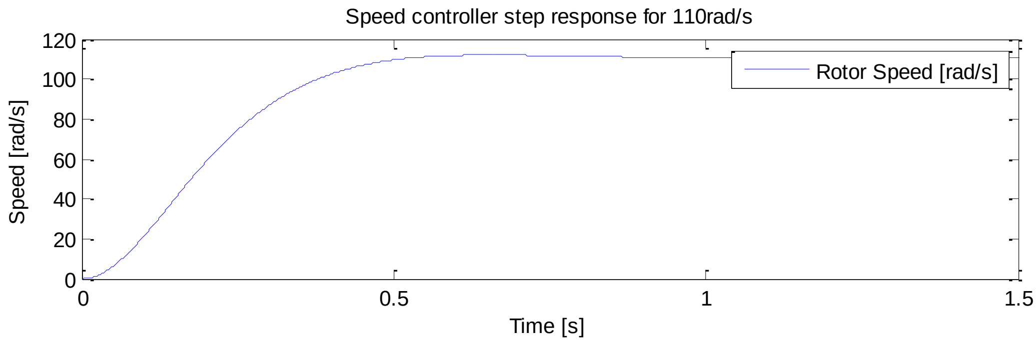

Fig.5. Speed controller step response

Description	Value
Stator inductance L_s	0.47H
Rotor inductance L_r	0.47H
Mutual inductance L_{sr}	0.44H
Leakage factor σ	0.12
Stator resistance R_s	0.8Ω
Rotor resistance R_r	3.6Ω
Moment of inertia D_m	0.06kg.m^2
Viscous damping constant R_m	0.04N.m.s
Number of pole pairs n_p	2

Table 1.Motor nominal parameters

4. CONCLUSION

This article presented a LPV control approach for nonlinear plants. The state space variable that caused nonlinearity was taken out as a varying parameter and two controllers were designed for vertices of this known (measured) parameter. Due to a vertex property [3] of the LPV current plant it is possible for a stable system to reduce infinite number of LPVs to solve to a finite number; in this case controller is designed for the two vertex plants. Each controller is computed by a convex solution of Bounded Real Lemma presented in [3]. Current controller was simulated for several values of varying parameter ω, magnitude of these changes is presented on motor behavior shown in Fig. 2. As shown in Fig. 3 LPV controller adjusts itself to varying values of ω. Step response values are similar for all variations (individual lines are overlapping in Fig. 3).From this simulation we can assume that impact on the closed loop performance is minimal. Achieved H_∞ quadratic performance of 0.79 guarantees that closed loop with LPV controller is robustly stable for all admissible values of ω. Final simulation of speed controller was performed with static controller which was designed for fast speed reference tracking and minimal overlap. Simple flux observer [1] was implemented to estimate α-β component of motor flux. This observer doesn't provide good flux tracking and estimation error can be observed in the case of change in motor parameters. This flux observer can be potentially replaced in order to improve the close loop stability and performance in the case of parameter change.

5. ACKNOLEDGMENT

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