MULTI-MODEL MODIFICATION OF MODEL PREDICTIVE CONTROLLER

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Abstract: Predictive controllers are nowadays a necessary part of industrial processes. The main reason for application of this kind of controllers is their specific attributes which provide many advantages in this area. In this paper we will design multi-model predictive controller and make a comparison with standard model predictive controller (MPC) in closed loop which contains plant with uncertain parameters. In the last part of the article we apply both predictive controllers in control circuit which contains the physical model.

Keywords: predictive control, receeding horizon, Kalman filter, artificial neural network, uncertainty, classification

1. INTRODUCTION

Predictive regulators came into consciousness between the 60's and the 70's of the last century. Their interesting properties caused that by the end of the 70's they were already implemented in industry. At that time there were already some modifications of predictive regulators and over time there are more and more. All of the predictive algorithms work on a similar principle.

In this article we will deal more closely whit the control loop which contains MPC (standard and modified) with state space model and a system with uncertain parameters. At the beginning we obtain a mathematical description and we state the uncertainties of the system. As the model of the system is a key requirement for the function of the MPC, we will determine the behaviour of the system in limiting areas. We will carry out the comparison on basis of the integral criteria (quadratic and IAE). In the end we will verify the acquired results on the physical model.

2. MODEL DESCRIPTION

To verify our work we have chosen a model of air-tunnel. This contains the proper air-tunnel which has at the entrance a ventilator attached which has the function of an actuator (controlled by PLC B&R in a range 0÷10V with 12-bit D/A converter). This ventilator drives air into the air-tunnel. The driving ventilator exhibits a 5÷8 % dead zone at the initial point of the coordinates. At the end of the tunnel there is a second ventilator which has the function of air-flow sensor and this linear motion is converted into rotational. By means of optoelectronic sensor the rotational motion is converted to electric signal it is further processed by additional electronics which sends this processed signal to 12-bit A/D converter in PLC. The PLC is connected to PC where runs the program MATLAB-Simulink by which the whole task is carried out. On the basis of experience with the model we will expect the order of the mathematical model equal to 3rd and its approximate transfer function is:

$$G(s) = \frac{K}{a_3 s^3 + a_2 s^2 + a_1 s + 1} \tag{1}$$

Parameters of the transfer function $G(s)$ are assigned from the system step response by means of searching minimum of quadratic criterion. The identification is realized for operation range 3÷4 V. The nominal values of the parameters are following: $K = 1{,}51$; $a_1 = 4{,}55$; $a_2 = 4{,}79$; $a_3 = 1{,}75$. These parameters can under the influence of the environment change in range ±10 %.

We use the MPC which contains discrete state space model in controllable canonical form [5]:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}\,\boldsymbol{x}_k + \boldsymbol{B}\,u_k \\ y_k &= \boldsymbol{C}\,\boldsymbol{x}_k \end{aligned} \tag{2}$$

For the reason of conditions

$$\boldsymbol{y} = \boldsymbol{w} \text{ and } \boldsymbol{\Delta u} = 0 \tag{3}$$

we modify the state space model by this way [2]:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{x}_{\boldsymbol{k+1}} \\ \boldsymbol{u}_{\boldsymbol{k}} \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & I \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_k \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} \\ I \end{bmatrix} \Delta u_k \\ y_k &= \begin{bmatrix} \boldsymbol{C} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_k \\ u_{k-1} \end{bmatrix} \end{aligned} \tag{4}$$

where Δu_k is increment of the control action.

3. CONTROLLER

In this work we use standard MPC with state space model representation which uses for state estimation discrete Kalman filter. The principle scheme of standard control loop depicts Fig. 1 (left). The modified MPC contains three state space models (Fs1 – worst case model, Fs2 – model with nominal parameters and Fs3 – model with slowest dynamic) with three Kalman filters and there are running optimizations for each of cases. This controller contains also neural network classificator and its structure depicts Fig. 1 (right). Both controllers use the receding horizon strategy.

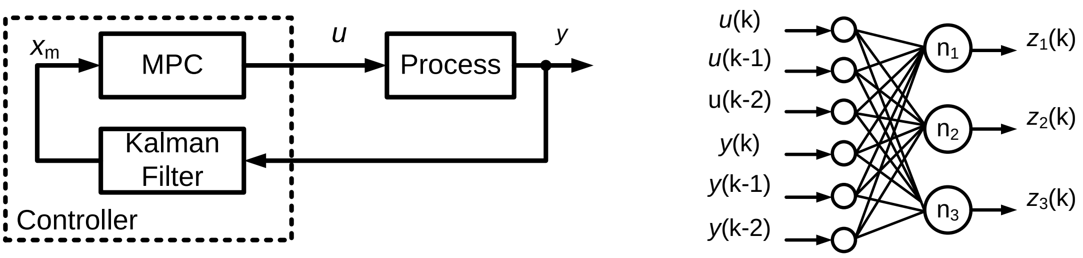

Figure 1: Control loop (left) and the neural network classificator (right)

3.1. MODEL PREDICTIVE CONTROL

The algorithm of MPC uses the principle of minimization of the following criterion [1]:

$$\boldsymbol{J} = [\boldsymbol{w} - \boldsymbol{y}]^{\boldsymbol{T}} [\boldsymbol{w} - \boldsymbol{y}] + \boldsymbol{\Delta u}^{\boldsymbol{T}} \boldsymbol{R}\, \boldsymbol{\Delta u} \tag{5}$$

Where $\boldsymbol{y}$ is acquired by this form:

$$\boldsymbol{y} = \boldsymbol{P}_{\boldsymbol{y}} \boldsymbol{x}_{\boldsymbol{m}} + \boldsymbol{H}_{\boldsymbol{y}} \boldsymbol{\Delta u} \tag{6}$$

Vector $\boldsymbol{w}$ is reference vector and $\boldsymbol{y}$ is vector of predicted system output. Construction of matrix $\boldsymbol{P}_{\mathrm{y}}$ and $\boldsymbol{H}_{\mathrm{y}}$ is described in [2].

The basic criterion can be reformulated into this form:

$$\boldsymbol{J} = [\boldsymbol{w} - \boldsymbol{P}_{\boldsymbol{y}} \boldsymbol{x}_{\boldsymbol{m}} - \boldsymbol{H}_{\boldsymbol{y}}\, \boldsymbol{\Delta u}]^{\boldsymbol{T}} [\boldsymbol{w} - \boldsymbol{P}_{\boldsymbol{y}} \boldsymbol{x}_{\boldsymbol{m}} - \boldsymbol{H}_{\boldsymbol{y}}\, \boldsymbol{\Delta u}] + \boldsymbol{\Delta u}^{\boldsymbol{T}} \boldsymbol{R}\, \boldsymbol{\Delta u} \tag{7}$$

We can obtain the minimization (7) by its derivation according to $\boldsymbol{\Delta u}$ and subsequently making the gradient of $\boldsymbol{J}$ equal to zero. The result of this is an equation (8) which is used for computation of the future control action increments.

$$\boldsymbol{\Delta u} = \left[\boldsymbol{H}_y^T \boldsymbol{H}_y + \boldsymbol{R}\right]^{-1} \boldsymbol{H}_y^T \left(\boldsymbol{w} - \boldsymbol{P}_y \boldsymbol{x}_m\right) \tag{8}$$

Predictive controller brings in one advantage - different approaches to restrictions according to the requirements on the controlled system. These requirements can hold out to restricting the input to the system or the output of the system. The requirements for the restriction of the input to the system will be sufficient for our task. Constraining conditions are in form:

$$\boldsymbol{R}\,\boldsymbol{\Delta u} \le \boldsymbol{c} \tag{9}$$

3.2. KALMAN FILTER

Necessary part of the MPC which uses state space model is the state observer. In this application we use the Kalman filter for state estimation.

The algorithm of Kalman filter is described by this way:

$$\begin{aligned} \hat{\boldsymbol{x}}_k^- &= \boldsymbol{A}\,\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{B}\,u_k \\ \boldsymbol{P}_k^- &= \boldsymbol{A}\,\boldsymbol{P}_{k-1}\,\boldsymbol{A}^T + \boldsymbol{Q} \end{aligned} \qquad \begin{aligned} \boldsymbol{K}_k &= \boldsymbol{P}_k^- \boldsymbol{H}^T \left(\boldsymbol{H}\,\boldsymbol{P}_k^- + \boldsymbol{R}\right)^{-1} \\ \hat{\boldsymbol{x}}_k &= \hat{\boldsymbol{x}}_k + \boldsymbol{K}_k \left(y_k - \boldsymbol{H}\,\hat{\boldsymbol{x}}_k^-\right) \\ \boldsymbol{P}_k &= \left(\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}\right)\boldsymbol{P}_k^- \end{aligned} \tag{10}$$

On left side is the “predictive” part and right side describes the “correction” part. The basics and examples of applications about the Kalman filter theory is described in details in [3].

3.3. NEURAL NETWORK CLASSIFICATOR

The classificator contains three neurons with back-propagation learning algorithm [4]. In first step we learnt separately each neuron for the response of the model output. Neuron n_1 is trained for detecting output of model Fs1, n_2 for model Fs2 and n_3 for model Fs3. In second step the plant output is adduced to acquire the weighting coefficients v_q (q = 1,2,3).

$$\begin{aligned} \Delta w_{ij}(t) &= \eta \delta_i(t) y_j(t) + \mu \Delta w_{ij}(t-1) \\ w_{ij}(t+1) &= w_{ij}(t) + \Delta w_{ij}(t) \end{aligned} \tag{11}$$

Weighting coefficients are maped to interval (0, 1) by Gaussian function:

$$v_q(z) = e^{-\left(1-z_q\right)^2} \tag{12}$$

Expression $1 - z_q$ reflects “classification” error. Form (14) is weighted mean and its result is final action control increment which is added to a plant input.

$$\Delta u = \frac{\sum_{q=1}^{3} v_q \Delta u_q}{\sum_{q=1}^{3} v_q} \tag{13}$$

4. EXPERIMENTAL RESULTS

In this section there are results from Simulink environment and measurement on physical model. There is a comparison between the standard MPC and the modified MPC.

4.1. MATLAB SIMULATION

The control loop in Simulink environment contains MPC (standard or modified), plant with uncertain parameters, and 12-bit A/D and D/A converter. The control loop does not contain any noise.

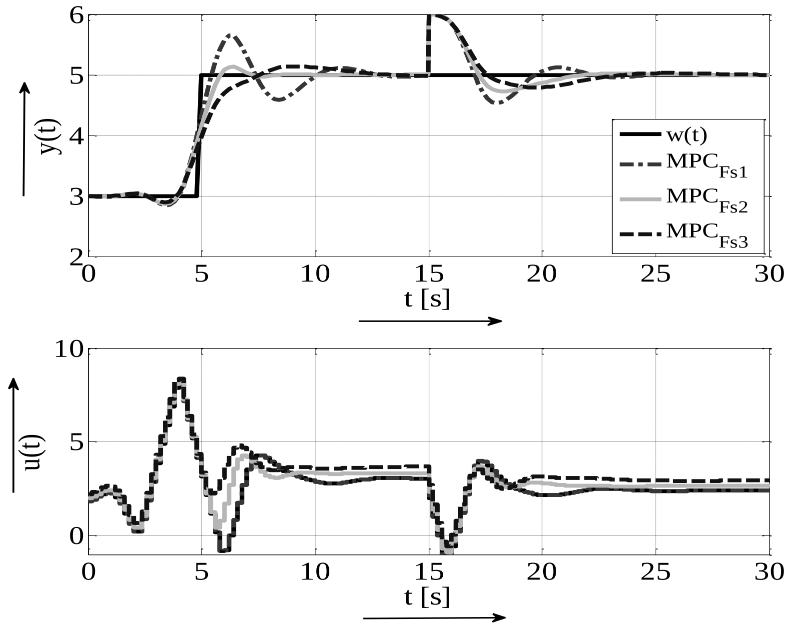

Figure 2: Control process with standard MPC controller in the control loop.

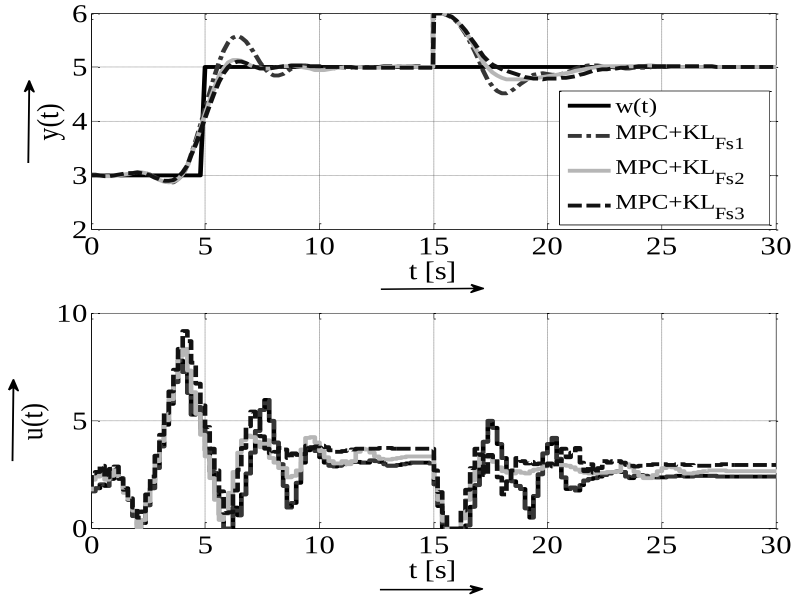

Figure 3: Control process with modified MPC controller in the control loop.

	Fs1		Fs2		Fs3	
	IAE	Quadr.	IAE	Quadr.	IAE	Quadr.
MPC	71,2	102,1	52,6	95,5	64,5	102,6
MPC+KL	64,8	97,5	55,4	96,0	55,6	100,5

Table 1: The results of integral criteria.

4.2. RESULTS OBTAINED ON PHYSICAL MODEL

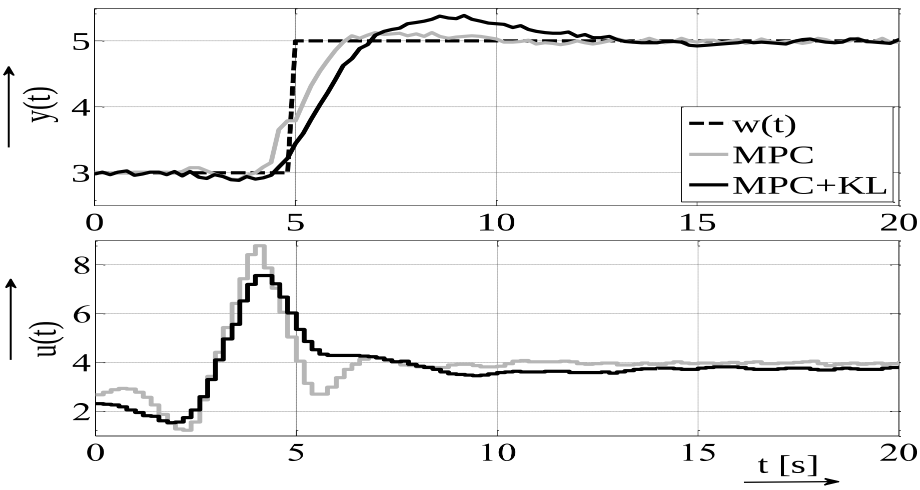

Figure 4: Control process on air-tunnel model.

4.3. FUTURE IMPROVEMENTS

For better function it is necessary to improve the MPC controller with neural network classificator. The improvements can be following: Noise robustness for classification, because the noise deforms results of classification. Optimality, because the form (14) causes suboptimality of modified controller. Optimization of computing, because there are high demands for computer performance.

5. CONCLUSION

Our task was to design the multi-model predictive controller. The modified controller contains three state space models with Kalman filters and neural network classificator for computing weighting coefficients. Practical results are in the section 4. In the Simulink environment the modified MPC controller ($\boldsymbol{R_1} = \boldsymbol{I} \cdot 0.001$; $\boldsymbol{R_2} = \boldsymbol{I} \cdot 0.007$; $\boldsymbol{R_3} = \boldsymbol{I} \cdot 0.0006$) was better than the standard MPC controller ($\boldsymbol{R} = \boldsymbol{I} \cdot 0.007$) for border cases (Fs1, Fs3). In the nominal case (Fs2) got better results the standard controller. It caused form (14), because the modified MPC controller was suboptimal and there was implicit control action for border cases. The results of the physical model indicate that the modified MPC work not well and noise causes problems in classification.

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