STOCHASTIC AND DETERMINISTIC ADAPTATION ALGORITHMS OF EQUALIZATION PROCESS IN DIGITAL COMMUNICATION SYSTEMS

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Abstract: The equalization is a process which uses the adaptation algorithm for compensating the undesirable influence of a digital channel on useful signal. The paper describes two main approaches of solving the equalization problem. The first approach is based on Wiener filter theory which deals with the statistical properties of underlying signals. The second approach is derived from the deterministic point of view. On the basis of particular algorithms implementation we found the factors which affect the quality of equalization.

Keywords: Equalizer, adaptation algorithm, Least-mean-square Algorithm (LMS), Recursive Least-square Algorithm (RLS)

1. INTRODUCTION

The problem of equalization consists in finding the filter parameters which minimize the distortion of the signal after passing through the channel.

The aim of this work is to simulate the existing adaptation algorithms and find the factors which affect the quality of equalization.

2. EQUALIZER

The equalizer is an adaptive filter which uses the adaptation algorithm to compensate the influence of the unknown system on the useful signal by minimizing the difference between the adaptive filter output and the delayed test signal.

The adaptation algorithm plays very important role in performance of equalizer. There are two groups of algorithm: stochastic and deterministic. We distinguish them according the approach of minimizing the performance function.

2.1. STOCHASTIC APPROACH

The stochastic approach is based on Wiener filter theory and uses mean-square error (MSE) as a performance function [2]:

$$\xi = \mathrm{E}[e(n)^2], \tag{2.1}$$

where the estimation error $e(n)$ is

$$e(n) = d(n) - y(n). \tag{2.2}$$

The minimizing of MSE requires a certain statistical evaluation of input signal. Taking the expectation and noting that $y(n) = x^T(n)w$ we can rewrite (2.1) as

$$\xi = \xi_{min} + (\mathbf{w} - \mathbf{w_0})^T \mathbf{R} (\mathbf{w} - \mathbf{w_0}), \tag{2.3}$$

where ξ_{min} is minimal MSE in case of optimal state of the equalizer, **R** is an autocorrelation matrix of input signal, $\mathbf{w}$ and $\mathbf{w_0}$ is a tap weight vector of adaptive filter during the adaptation process and in optimal state correspondingly.

The performance surface of (2.3) is $(N+1)$ dimensional hyperboloid, where N is length of adaptive filter. The shape of such hyperboloid is determined by the autocorrelation matrix **R**. Using the theory of eigenvalues we can rewrite **R** as

$$\mathbf{R} = \mathbf{Q\Lambda Q^T}, \tag{2.4}$$

where **Q** is the matrix of eigenvectors of **R** and **Λ** is the diagonal matrix of eigenvalues of **R**. Since the eigenvalues spread of correlation matrix of stochastic process is related to power spectral density of this process we can say that performance of stochastic algorithms depends on the power spectral density of input signal.

Another important factor of stochastic algorithms is the size of the step-size parameter. The convergence speed of $w(n)$ to the optimum solution $\mathbf{w_o}$ depends on the size of the step-size parameter. Also the step-size parameter affects the stability of algorithm. It was proved in [1] that the step-size parameter should be in the range:

$$0 < \mu < \frac{1}{\lambda_{max}}, \tag{2.5}$$

where λ_{max} is the maximum eigenvalue of autocorrelation matrix **R** and μ is the step-size parameter.

The most widely used algorithm of this group is Least-mean-square Algorithm (LMS).

2.2. DETERMINISTIC APPROACH

Deterministic approach is based on the method of least square. In contrast to Wiener filter theory the method of least square approaches the problem of filter optimization from a deterministic point of view. This method uses the sum of weighted error squares for the given data as a performance function:

$$\text{?}(\mathrm{n}) = \sum_{\mathrm{k}=1}^{\mathrm{n}} \sigma_n(k) e_n^2(k), \tag{2.6}$$

where ?(n) is performance function, $e_n(k)$ are the samples of error estimate at ns iteration, and $\sigma_n(k)$ is weighting function [1].

Adaptation algorithm is used to minimize this function. This is the deterministic optimization which is based on the observed data. The learning curve of the algorithm is represented as

$$\xi_{n-1}(n) = \xi_{min} + 1 - \frac{1-\lambda}{1+\lambda} * \frac{1+\lambda^{n-1}}{1-\lambda^{n-1}} * N\, \xi_{min}, \tag{2.7}$$

where ξ_{min} is minimal MSE. We see that in contrast to stochastic approach the convergence behavior doesn't depend on the eigenvalues **R** or in other words it is independent of power spectral density of input signal. But the convergence behavior depends on the parameter λ which affects the number of previous samples participated in each iteration of the algorithm.

2.3. SIMULATION

To prove statements above we are going to simulate two models of channel with the equalizer. The equalizer will be used to repair the signal after passing through the channel. In the first case the input signal will be close with white sequence[1], in the second case the input signal will have the spectrum concentrated in a narrow range of frequencies. As adaptive algorithm we will use LMS algorithm meaning the stochastic algorithm and RLS algorithm meaning the deterministic algorithm. The results of simulation are shown in the figure 1.

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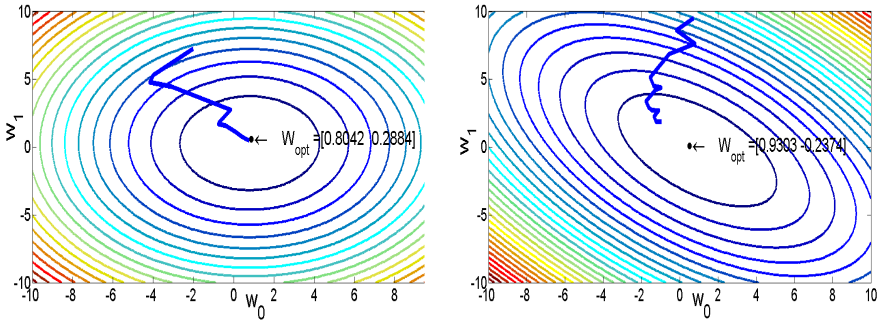
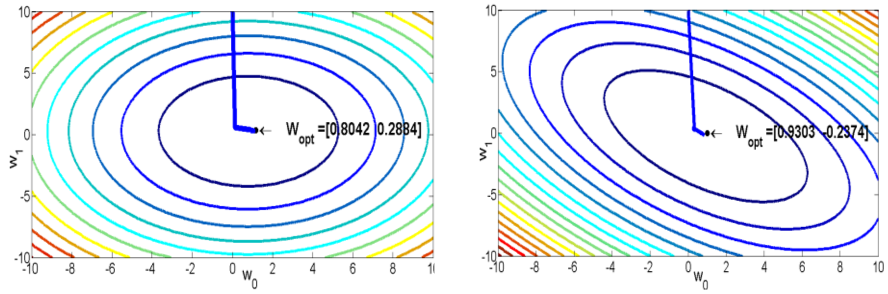
[1] White sequence means that power spectrum is flat across the whole range of frequencies



a)



b)

Figure 1 The trajectories of the tap weights of equalizer during the process of adaptation in case of using white and highly coloured input sequence

The figure 1 a) represents performance surface of the LMS algorithm in case of two-tap transversal filter. For convenience the surface is shown in 3-dimensional Euclidian space which axes are two independent taps of the filter and the function ξ. The optimal state of the equalizer is indicated by arrow. In both cases the number of iteration is the same. As we see from the figure in the first model (left image) the convergence is almost complete, in the second model (right image) more iterations are require before they converge to the minimum point.

The figure 1 b) represents adaptation process of RLS algorithm. In both cases convergence is almost complete independently on input signals. We see from the number of kinks on the curve that the speed of convergence to the optimal state is less than in case of using LMS algorithm.

3. CONCLUSION

We can say that performance of stochastic algorithms is highly dependent on the power spectral density of the filter input. Properly when the filter input is close with the white sequence the LMS algorithm converges very fast. However, when the certain frequency bands are not well excited the convergence is slower. The step-size parameter plays a significant role in controlling the performance of the LMS algorithm. On the one hand the convergence speed of algorithm changes in proportion to its step-size parameter. But on the other hand a small step-size parameter has to be used to achieve the stability of the algorithm. The performance of Deterministic algorithm is independent on the power spectral density, and this is the main advantage of it.

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