BIFURCATION STRUCTURE OF DC MICRODRIVES

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ABSTRACT

This study deals with bifurcation analysis of direct-current microdrives. It contains the analysis of both electrical and non-electrical quantities (tension, current, speed, torque, etc.) depending upon the change of one or more of the parameters (supply voltage, temperature, friction coefficient, frequency, etc.). That is, electrical drives can be considered as complex non-linear dynamic systems with variable parameters. These parameters can – on certain system conditions – show certain irregularities in the behaviour of observed quantities.

1. INTRODUCTION

Bifurcation is all around us: in nature, medicine, mathematics, geography, etc. This paper deals with the kind of bifurcation we can find in electromechanics; to be specific, it deals with the analysis of bifurcation diagrams in a system of a DC motor with a regulator.

Bifurcation images belong to a set of fractals. Fractals are images similar to each other, the shape of which seems complicated at first glance, but in fact they are generated by a repeating row of images created with simple rules. Bifurcation is a name for such change of an output parameter (quality) of the system which results in separation of the original trajectory of the parameter development to several new structures of various levels (owing to even a slight change of one or more of the input data).

2. BIFURCATION

The process of a general bifurcation diagram can be exemplified, as seen in Fig. 1. Detailed scrutiny will reveal areas where trajectory separation occurs (the so-called critical points). The bifurcation diagram also shows evident regularities – restricting limits and „white windows“, where under certain conditions an „order“ in the otherwise chaotic design can occur.

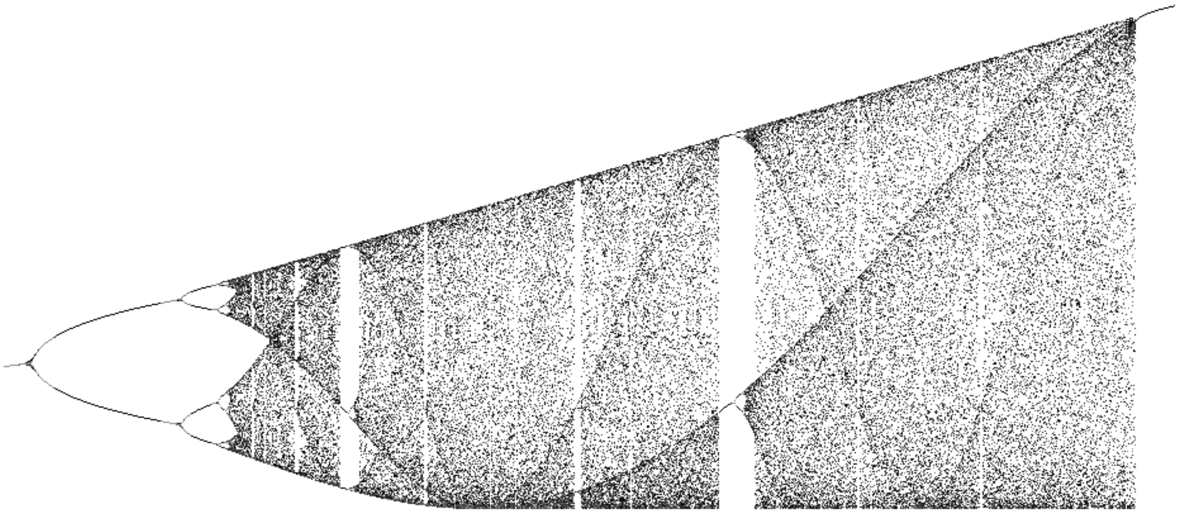

Fig. 1: General bifurcation diagram

Practical measurement can turn out problematic when searching for bifurcation points; the swing of the observed quantity may sometimes be so little that it vanishes because of the small accuracy of the meter. In general, it is possible to find 2 or 3 bifurcation points during a practical measurement with the trajectory splitting up to 4 or 8 levels.

The trajectory split area (i.e., the bifurcation) is not parametrically infinitely short. Gradual trajectory splits can be used for a hopeful prediction of the system‘s state.

3. COMPUTER SIMULATION

Carrying on a system diagnosis on a PC and seeking out bifurcation points can be done most easily by the help of a mathematical model of a DC motor with a PI speed regulator. This DC model stems from the voltage equation for a DC engine with separate excitation (a permanent magnet, for instance).

$$u_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + c \cdot \phi \cdot \omega \qquad (1)$$

$$c \cdot \phi \cdot i_a - Mz - B \cdot \omega = J \cdot \frac{d\omega}{dt} \qquad (2)$$

,where:

u_a	…	supply voltage [V]	$c \cdot \phi$	…	speed constant [$V \cdot s$]
i_a	…	current [A]	ω	…	speed [s^{-1}]
R_a	…	winding resistance [Ω]	Mz	…	torque [$N \cdot m$]
L_a	…	winding inductance [H]	B	…	friction coefficient [$N \cdot m \cdot s$]
t	…	time [s]	J	…	rotor inertia [$kg \cdot m^2$]

3.1. Analysis parameters

In the designed bifurcation analysis model, two output quantities were observed: one being the output engine speed and the second being the demanded current from the power supply unit. It is these two quantities that are crucial for the motor control system. The output system speed must be stable, so that the exuberant vibration of the motor itself or the connected appliance doesn‘t happen. Far too high demanded current fluctuation may result in disturbances and/or exuberant dynamic straining of the power supply system.

The other quantities (like the proportional or integration amplification of the regulator speed, the amplification of the speed and current sensor, the regulator working frequency, etc.) are used as the system's input parameters. Changing of these parameters may be used for simulating various conditions of the entire system's behaviour. The output value diagrams are depicted subsequently in relation with the change of the corresponding parameter.

3.2. Bifurcation diagrams

The easiest way of creating a bifurcation diagram is by changing only one parameter in a proper range and considering the other stable. The output quantities are depicted in relation with the changed parameter.

The position and frequency of bifurcation points don't depend solely on one parameter, but on the whole scale figuring in the simulation.

In Fig. 2, there is an evident bifurcation diagram of the supply current I_a, while the changed parameter is the proportional amplification of the speed regulator $K\omega_p$. The figure shows that if the demanded current is to be in the stable area, the value of the proportional amplification of the speed regulator must range from cca 0.5 to 4 .

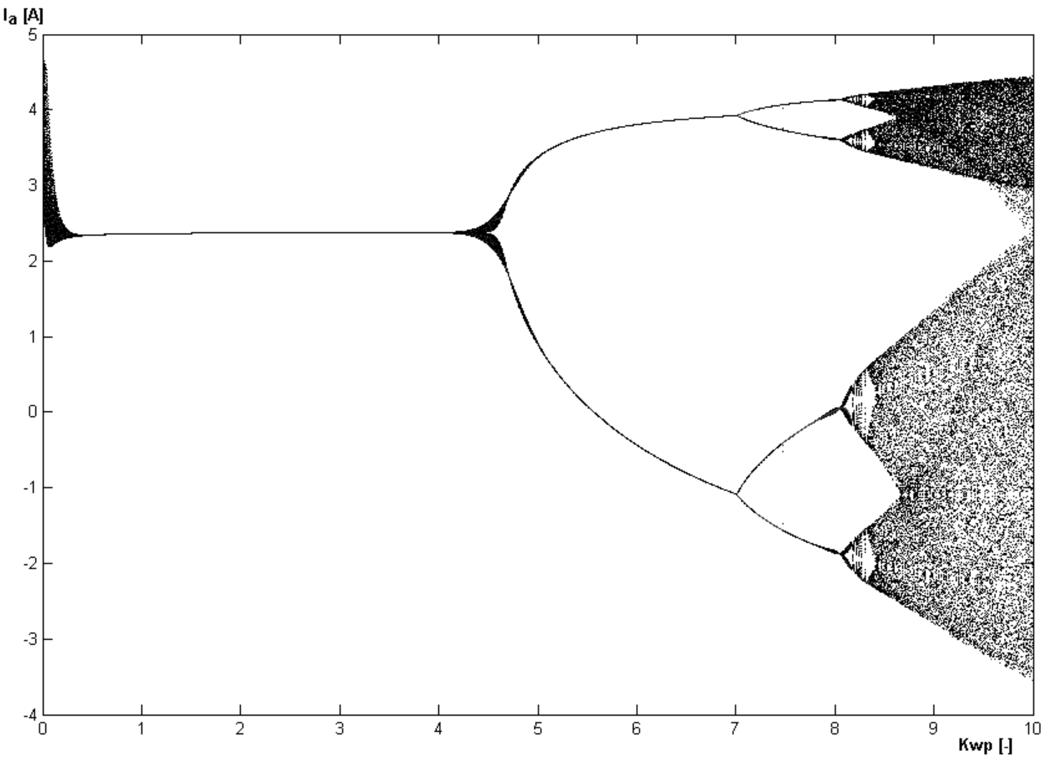

Fig. 2: Bifurcation diagram of the supply current Ia, the parameter being $K\omega_p$

In the speed bifurcation diagram (Fig. 3), it is evident that the value of $K\omega_p$ has a significant impact on the output speed. That's why it is necessary for reaching the demanded speed value (that's 1200 rpm in our case) and keeping the regulator in the stable area to choose an amplification value equal to 4.

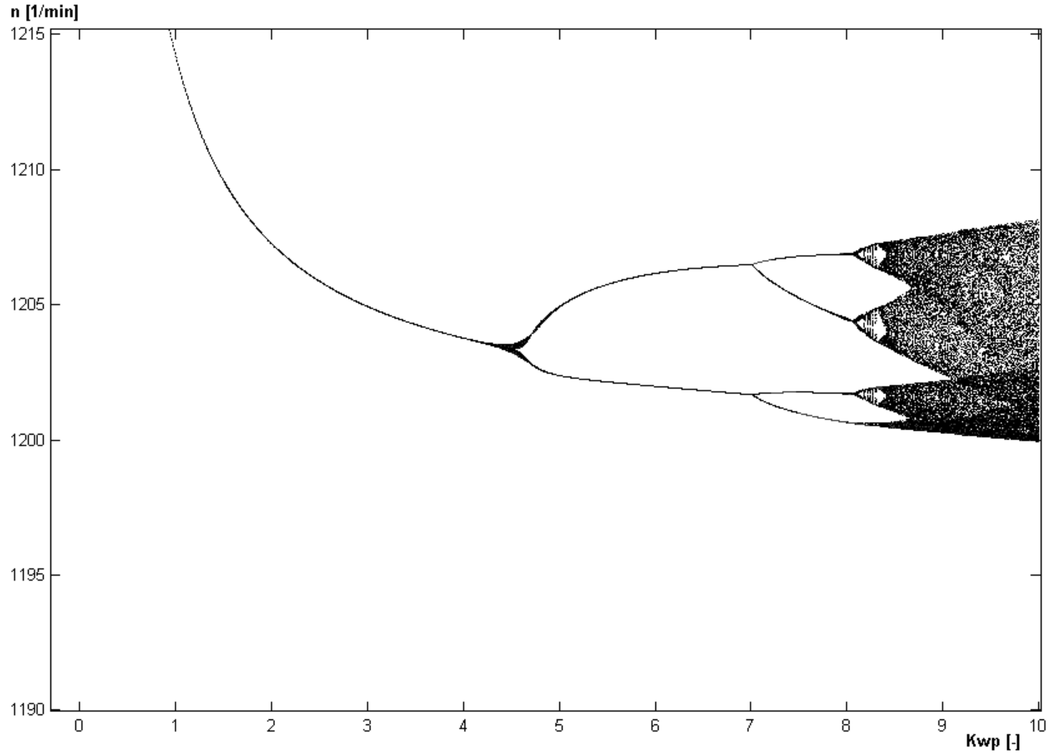

Fig. 3: Speed bifurcation diagram, the parameter being $K\omega_p$

3.3. Bifurcation analysis utilization

Bifurcation analysis can be used – among others – for designing and setting up the regulator parameters. The initial values can be determined for example by applying the optimization method. Using a row of successive simulations for the sweep of individual parameters will give us a series of bifurcation diagrams that allows us to re-set the regulator in order for the working point to be kept in the stable area as far as the current and the speed is concerned, while maintaining the demanded output values of the engine speed.

Similarly, it is possible to carry on the simulation of the regulator's behaviour while swinging other parameters, such as the amplification of the current or speed probe, the fluctuation of the regulator's frequency, or the supply voltage ripple.

4. PRACTICAL READING OF BIFURCATION

It is of course possible to observe bifurcation on real systems as well, not only on computer simulations. The problem with real measuring, though, usually lies in the sensitivity of used devices, because the swing between the levels may be smaller than what the device is capable of determining.

Fig. 4 shows the time behaviour of an observed quantity while gradually changing the directing parameter. This proves that bifurcation splitting doesn't occur fitfully; on the contrary, a gradual swing of the observed quantity occurs first. Fig. 5 depicts the progress between the second and the third bifurcation (the signal takes four values).

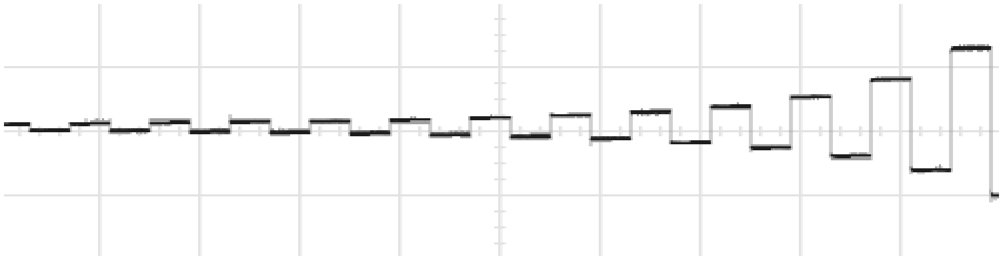

Fig. 4: The behaviour of the observed quantity while gradually changing the directing parameter

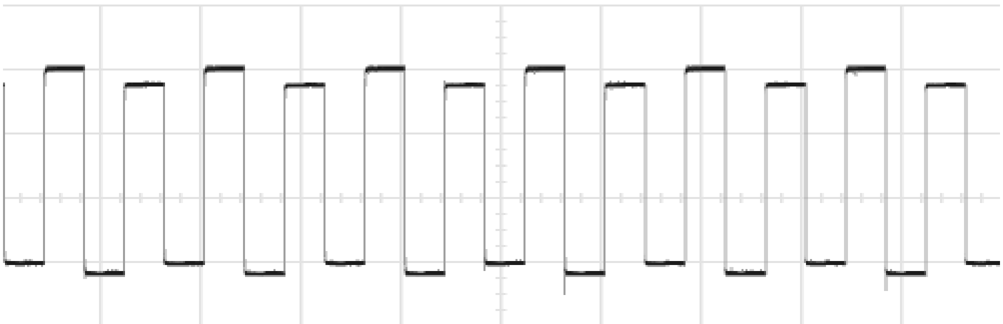

Fig. 5: The behaviour between the second and the third bifurcation

5. CONCLUSION

Bifurcation analysis of an electromechanical (mechatronic) system can be considered very hopeful when observing the changes of the parameters of a non-linear dynamic system, especially from the viewpoint of the predictability of failure states (diagnostics).

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