THE ELECTRIC CAR DYNAMICS ANALYSIS

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ABSTRACT

The article deals with the analytic solution of the electric car acceleration. The solution was created in the general form as the analytic equations in the closed shape, and not with the help of numerical methods.

1. INTRODUCTION

The whole energy spending during the acceleration is very critical at the electric car drive supplied with the hydrogen fuel cell in the combination with the ultracapacitor. The reason of it is the relative small energy amount stored in the ultracapacitor. However, the ultracapacitor usage brings the great advantage which lies in the fact, that the hydrogen fuel cell can be designed to the average travelling power, and not to the peak power ($P_{av} : P_{peak} = 1 : 3$ obviously). The peak power at the accelerations and decelerations is covered from the ultracapacitor.

So, from the energy consumption point of view, it is useful to analyse the car acceleration very precisely in the general analytic form. These results can serve as input data for the basic drive design at different input conditions.

2. FRICTION FORCES

At the general solution of the acceleration it is necessary to consider all force types: gravitation force, dry and rolling frictions, viscous friction, aerodynamic friction.
At the dry and rolling friction the whole force F_F is *constant*, independent on the velocity

$$F_F = \xi g m . \quad (1)$$

The coefficient of the rolling friction $\xi = 0.02$ approximately.
Viscosity friction force F_V (e.g. in the gear box) depends on the *first* power of the velocity:

$$F_V = k_V v(t) , \quad (2)$$

where k_V is the viscosity coefficient.
For the aerodynamic pressure the equation $p = 1/2\ (\rho v^2)$ is valid, where $\rho = 1.29 \mathrm{kg/m^3}$ is the air mass density. Aerodynamic friction force depends on the *second* power of the velocity:

$$F_{\mathrm{A}} = \frac{1}{2} C_x \rho S v^2(t)\,, \tag{3}$$

where S is the area of the perpendicular frontal car projection, C_x is non-dimensional coefficient closed to number 0.4 which is dependent on the car shape.

3. VELOCITY CALCULATION

Initial preposition:

- The machine traction force F is kept constant in the velocity range $v = 0$ up to $v_{\max}$ duri the acceleration.
- Maximum traction force $F_{\max}$ is restricted by the physical way on the constant value (e.g. by the current limiting in the converter: $F_{\max} \approx T_{\max} \approx I_{\max}$). The value $F_{\max}$ is independent on the velocity in the all velocity range $v = 0$ up to $v_{\max}$.
- Maximum velocity $v_{\max}$ is restricted by the physical way on the constant value (e.g. by the maximal converter output voltage).
- In the equilibrium state for the limit velocity $v_{\lim}$ the inequality $v_{\lim} \le v_{\max}$ is valid even though at $F = F_{\max}$.

Under these preposition the following motion equation is valid:

$$F = m\frac{\mathrm{d}v(t)}{\mathrm{d}t} + mg\sin\alpha + \xi\, g\, m + k_{\mathrm{V}} v(t) + \frac{1}{2} C_x \rho S v^2(t)\,, \tag{4}$$

The α angle corresponds to the set inclination. It is deals with the differential equation of the first degree which will be arranged into following form:

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = -\frac{1}{2m} C_{\mathrm{x}} S \rho v^2(t) - \frac{k_{\mathrm{V}}}{m} v(t) + \frac{F - mg\sin\alpha - \xi\, g\, m}{m}\,. \tag{5}$$

The following constants a, b, c, τ will be introduced for the simpler notation:

$$a = \frac{C_{\mathrm{x}} S \rho}{2m} \ne 0\,, \quad b = \frac{k_{\mathrm{V}}}{m}\,, \quad c = -\frac{F - mg\sin\alpha - \xi\, g\, m}{m}\,, \quad \frac{1}{\tau} = \sqrt{b^2 - 4ac}\,. \qquad (6), (7), (8), (9)$$

The constant a must not equal to zero, in this case the solution would be wholly different, and simpler. After the constant putting, the equation must be transform into the separate form

$$\mathrm{d}t = -\frac{\mathrm{d}v(t)}{a v^2(t) + b v(t) + c}\,, \quad \mathrm{d}t = -\frac{\mathrm{d}x}{a x^2 + b x + c} \qquad (10a), (10b)$$

From formal reasons only the remarking $v(t) \to x$ will be used. The separate equation can be directly integrated. The solution method is strongly depended on the sign of the determinant ($b^2 - 4ac$). At the acceleration certainly is valid

$$F - mg\sin\alpha - \xi\, g\, m \ge 0\,. \tag{11}$$

This is the reason why the coefficient c is negative. For that reason the term ($b^2 - 4ac$) is positive, hence the asked integral will have the following form, [1]:

$$t=-\int_{v_0}^{v(t)}\frac{dx}{ax^2+bx+c}=-\tau\left[\ln\left|\frac{2ax+b-\frac{1}{\tau}}{2ax+b+\frac{1}{\tau}}\right|\right]_{v_0}^{v(t)}. \tag{12}$$

The bottom integration boundary has the significance as the initial velocity v_0 from which the acceleration starts. The top integration boundary has the significance of the asked instantaneous velocity. After putting both of boundary, the bottom and top, we gain the term whose significance is the *whole acceleration time* from velocity v_0 to the actual velocity $v(t)$:

$$t=\tau\left[\ln\left|\frac{2av_0+b-\frac{1}{\tau}}{2av_0+b+\frac{1}{\tau}}\right|-\ln\left|\frac{2av(t)+b-\frac{1}{\tau}}{2av(t)+b+\frac{1}{\tau}}\right|\right]. \tag{13}$$

The asked instantaneous velocity during the acceleration goes out directly from equation (13):

$$v(t)=\frac{1}{2a\tau}\frac{2av_0+b+\frac{1}{\tau}+\left(2av_0+b-\frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}}{2av_0+b+\frac{1}{\tau}-\left(2av_0+b-\frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}}-\frac{b}{2a}. \tag{14}$$

The limit velocity v_{lim} in the steady state can be determined as the limit for $t\to\infty$:

$$v_{\mathrm{lim}}=\frac{1}{2a\tau}-\frac{b}{2a}=\sqrt{2\frac{F-mg\sin\alpha-F_{\mathrm{F}}}{k_{\mathrm{A}}S\rho}+\left(\frac{k_{\mathrm{V}}}{k_{\mathrm{A}}S\rho}\right)^2}-\frac{k_{\mathrm{V}}}{k_{\mathrm{A}}S\rho} \tag{15}$$

The instantaneous acceleration will be gain as the differentiation of the equation (14):

$$a(t)=\frac{\mathrm{d}v(t)}{\mathrm{d}t}=\frac{\left(2av_0+b-\frac{1}{\tau}\right)\cdot\left(2av_0+b+\frac{1}{\tau}\right)\cdot e^{-\frac{t}{\tau}}}{a\tau^2\left[2av_0+b+\frac{1}{\tau}-\left(2av_0+b-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}}\right]^2}. \tag{16}$$

Let us note that the analytic solution given by equations (13) up to (16) is wholly general.

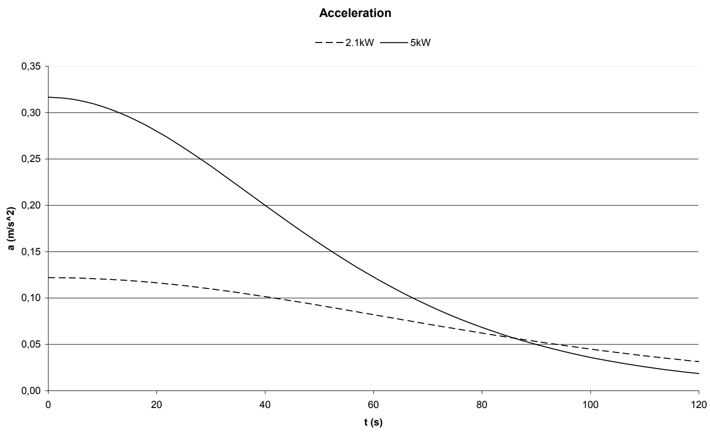

Figure 1: Acceleration.

4. ENERGY RELATIONS AT THE ACCELERATION

On the presumption that the slip does not appear in the gearbox nor at the wheel-road touch, the instantaneous mechanical power on the motor shaft is given by the equation

$$p(t) = F(t)\,v(t) = F\left(\frac{1}{2a\tau} \frac{2av_0 + b + \frac{1}{\tau} + \left(2av_0 + b - \frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}}{2av_0 + b + \frac{1}{\tau} - \left(2av_0 + b - \frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}} - \frac{b}{2a} \right) \tag{17}$$

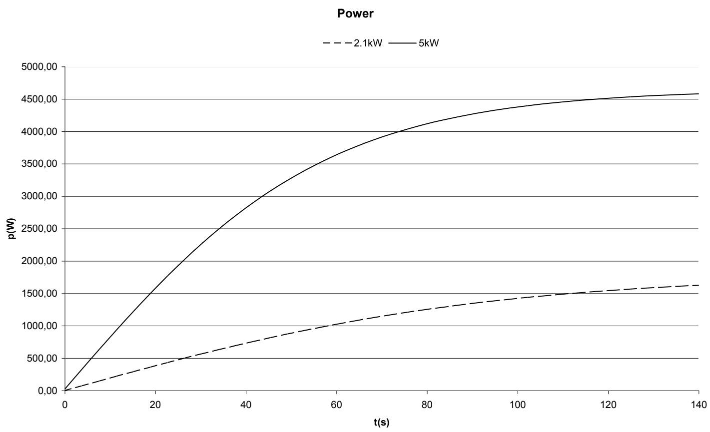

Figure 2: Power.

Let us emphasise that $F(t)$ is the equivalent force corresponding to the torque on the motor shaft.

The constant power in the steady state can be determined as the limit for $t \to \infty$:

$$P_{\lim} = \frac{F}{2a}\left(\frac{1}{\tau} - b\right) \tag{18}$$

The energy which is spent during the acceleration in the time interval 0 up to t:

$$W(t)=\int_0^t p(t)\,\mathrm{d}t = F\int_0^t\left(\frac{1}{2a\tau}\frac{2av_0+b+\frac{1}{\tau}+\left(2av_0+b-\frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}}{2av_0+b+\frac{1}{\tau}-\left(2av_0+b-\frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}}-\frac{b}{2a}\right)\mathrm{d}t \tag{19}$$

After solving:

$$W(t)=\frac{F}{2a}\left\{\left(\frac{1}{\tau}-b\right)t+2\ln\left[\frac{\tau}{2}\left(2av_0+b+\frac{1}{\tau}-\left(2av_0+b-\frac{1}{\tau}\right)\mathrm{e}^{-\frac{t}{\tau}}\right)\right]\right\} \tag{20}$$

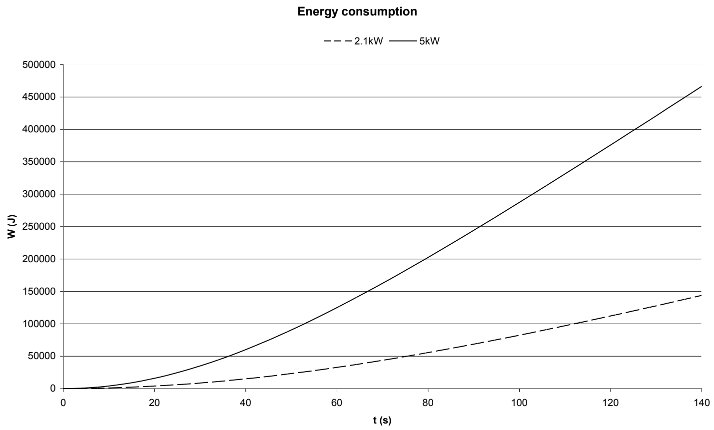

Figure 3: Energy consumption.

The second term on the right side of the equation (20) is negative, and it says how much is the whole energy smaller in the acceleration case when comparing to the energy spent in the same time interval but in the limit steady state at the constant power $P_{\mathrm{lim}} = Fv_{\mathrm{lim}}$ (videlicet, during the acceleration is valid inequalities $v(t) \leq v_{\mathrm{lim}}$ hence $p(t) \leq P_{\mathrm{lim}}$).

5. CONCLUSION

In the paper was shown the analytic solution of the electric car acceleration. The introduced example shows that the usage of these analytic results is very simple.

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