FINDING A RESONANCE FREQUENCY OF A SPECIAL TYPE OF CAVITY RESONATOR

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ABSTRACT

The article deals with finding a resonance frequency of a simple cylindrical cavity resonator for a certain geometrical structure of electromagnetic field. Both analytical and numerical solutions are included. The latter uses a frequency domain finite element method (FDFE). Results given by both methods are compared.

1 INTRODUCTION

Due to profound property changes during the process of designing resonance circuits with microwave resonance frequencies, inductors and capacitors cannot be used. [2] suggests a cavity resonator as a simple solution. We deal with a special type with vacuum-filled cavity and ideally conductive walls and try to find a resonance frequency of a specific resonator.

2 ANALYTIC SOLUTION

Analytic solution of a cylindrical resonator can be found in e.g. [2]. Using

$$c^2 \oint_{\Gamma_1} \vec{B} d\vec{s} = \frac{\partial}{\partial t} \oiint_{\Omega_1} \vec{E} d\vec{S} .$$

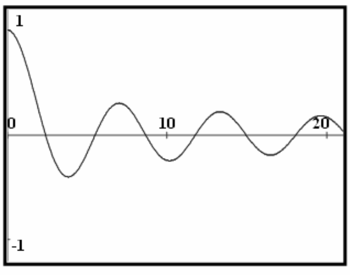

Fig. 1: *Function* J_0

we calculate magnetic field $\vec{B}$ from harmonic $E = E_0 e^{j\omega t}$. The out coming time-variable magnetic field causes an electric field. As a result the original electric field has to be “corrected”. This causes a need for “correction” of the magnetic field for more precise $\vec{B}$, etc. The electric field can therefore be written in the form of an infinite series, expressed by a zero-grade Bessel function of the first type J_0 (see Fig 1). The electric field in the resonator cavity therefore is:

$$E = E_0 e^{j\omega t} J_0\left(\frac{\omega r}{c}\right) \qquad (1.1)$$

where r is distance from the resonator axis. We use the second root of the given Bessel function J_0 ($x_2 = 5.52$) and compute a resonance frequency of a cylindrical resonator with $r = 87.85\,\mathrm{e}-3\,\mathrm{m}$. If the outer walls are in such a distance from the resonator axis that the electric field is neutral, (1.1) enables us to find the resonance frequency of the resonator:

$$f_0 = 5.52 \frac{c}{2\pi r} = 5.52 \frac{3 \cdot 10^8}{2\pi 87.85e-3} = 3.00\ GHz$$

3 NUMERICAL SOLUTION

We identify the positive direction of the z-axis with the direction of vector $\vec{E}$ identical with the resonator axis (see fig. 2). As [3] suggests, we substitute $E_z = E(x,y)e^{j\omega t}$ into a wave equation (regarding that the field intensity amplitude does not change in the positive direction of the z-axis). We get a wave equation:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k_0^2 E = 0 \qquad (2.1)$$

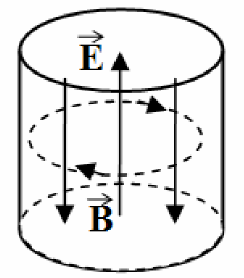

Fig. 2: *Cavity resonator*

with boundary conditions $\vec{n} \times \vec{E} = \vec{0}$ (tangential components of electric field intensity of an ideal conductor are zero) and $\vec{n} \times (\vec{\nabla} \times \vec{E}) = \vec{0}$ (change of components of the electric intensity vector in the direction of normal to an ideal magnetic conductor must be on such a surface zero). Instead of an exact distribution of intensity we substitute an approximate distribution $e(x,y)$.

$$\frac{\partial^2 e}{\partial x^2} + \frac{\partial^2 e}{\partial y^2} + k_0^2 e = R(x,y) \qquad (2.2)$$

$R(x,y)$ shows error of the approximate solution in the given points of the area in question. This function – residue – ceases to be a function of time for a steady harmonic state.

When discreting the analysed structure, use of 316 right-angled triangles (with side lengths adjusted to the shape of a circle) showed adequately precise. Considering symmetry of the resonator and homogeneity of the electric field in the direction of the z-axis, the analysed object can be simplified to a quarter of the resonator circular section and 79 elements (Fig. 3).

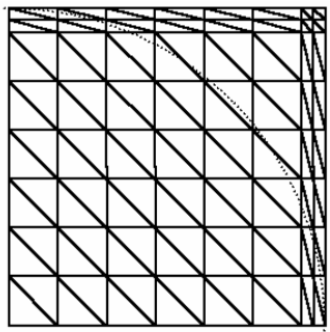
Fig. 3: *Discreting the structure*

For the space discreted structure we approximate the test field $e(x,y)$ by a piecewise linear function $e(x,y) = \sum_{m=1}^{M} e_m N_m(x,y)$, where e_m are the values of the test field in the triangle vertexes, $N_m(x,y)$ linear base functions. We substitute the piecewise linear approximation to (2.2) and using the weighted residue method we minimize the residue. Thus we have transformed PDE (2.2) for a continuous test field $e(x,y)$ into an algebraic equation with knot values of the test field as unknown scalar coefficients. If the number of weighting functions equals number of unknown knot values e_m, we get a solvable system of algebraic equations:

$$\mathbf{Se} + k^2\mathbf{Te} = \mathbf{0}$$

where **S**, **T** are matrices of known coefficients and **e** is the vector of unknown knot values of the test field. Solutions of this equation are represented by pairs [k^2, **e**] (see [1] and [3]).

4 COMPARISON

I have chosen such vectors **e**, for which the value corresponding to the distance of *r* from the resonator axis approximates zero. This condition results in $f_0 = 3.067$ GHz (as well as 1.311 GHz and several other values as FDFE was not "aware" that the second root of the Bessel function had been used in the previous solution and gave also results for other roots). $f_0 = 3.067$ GHz corresponds to the second root of J_0 and the electric field flow from Fig. 4 (middle) computed by MATLAB. Using the first root ($x_1 = 2.405$) and (1.1) we have:

$$f_0 = 2.405\frac{c}{2\pi r} = 2.405\frac{3\cdot 10^8}{2\pi 87.85e-3} = 1.307\ GHz$$

We would get the corresponding flow of electric field by cutting off the negative values of intensity in fig. 4 (left). Electric field inside the resonator changes its size not orientation.

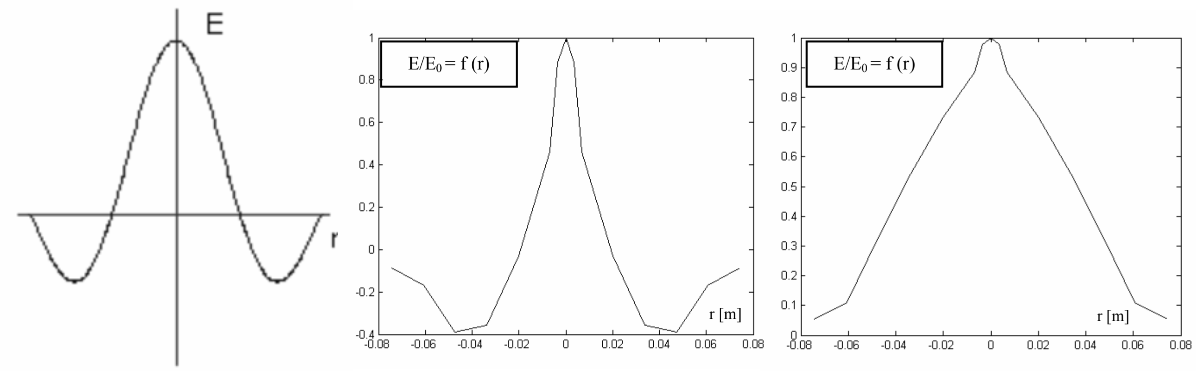

Fig. 4: *Comparison of results: analytic solution (left), electric field flow computed by MATLAB using FDFE for the second (middle) and the first (right) root of* J_0.

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